Fundamentals of Bayesian Epistemology 2

Arguments, Challenges, Alternatives

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Dutch Book Arguments

Chapter 8 presented the Representation Theorem Argument for probabilism. In its best form, this argument shows that any agent who satisfies certain preference axioms and maximizes expected utility assigns credences satisfying Kolmogorov's probability rules.[1] Contraposing, an agent who maximizes expected utility but fails to assign probabilistic credences will violate at least one of the preference axioms.

But why should rationality require satisfying the preference axioms? In Chapter 7 we argued that an agent who violates certain of the preference axioms (such as Preference Asymmetry or Preference Transitivity) will be susceptible to a money pump: a series of decisions, each of which is recommended by the agent's preferences, but which together leave her back where she started with less money on her hands. It looks irrational to leave yourself open to such an arrangement, and therefore irrational to violate the preference axioms.

While money pumps may be convincing, it's an awfully long and complicated road from them to probabilism. This chapter assesses a set of arguments that are similar to money pump arguments, but which constrain credences in a much more direct fashion. These arguments show that if an agent's credences violate particular constraints, we can construct a Dutch Book against her: a set of bets, each of which the agent views as fair, but which together guarantee that she will lose money come what may. Dutch Books can be constructed not only against agents whose credences violate Kolmogorov's probability axioms, but also against agents whose credences violate the Ratio Formula, updating by Conditionalization, or some of the other credal constraints we considered in Chapter 5.

Historically, Dutch Book Arguments were a very important motivation for probabilism. de Finetti (1937/1964) called Dutch-Bookable credence distributions "incoherent", and thought them rationally inconsistent.[2] When a Bayesian proposed a new constraint on credences, she would often vindicate that new requirement by showing that agents who violated it were susceptible to a Dutch Book.

In this chapter we will work through a variety of Bayesian norms, showing how to construct Dutch Books against agents who violate each one. We will

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then ask whether the fact that a Dutch Book can be constructed against agents who violate a particular norm can be turned into an argument that that norm is rationally required. After offering the most plausible version of a Dutch Book Argument we can find, we will canvass a number of traditional objections to that argument.

9.1 Dutch Books

Dutch Book Arguments revolve around agents' betting behavior, so we'll begin by discussing how an agent's credences influence the bets she'll accept. For simplicity's sake we will assume throughout this chapter that an agent assigns each dollar the same amount of utility (no matter how many dollars she already has). That way we can express bets in dollar terms instead of worrying about the logistics of paying off a bet in utils.

Suppose I offer to sell you the following ticket:

This ticket entitles the bearer to \$1 if *P* is true, and nothing otherwise.

for some particular proposition *P*. If you're rational, what is your fair price for that ticket—how much would you be willing to pay to possess it? It depends how confident you are that *P* is true. If you think *P* is a long shot, then you think this ticket is unlikely to be worth anything, so you won't pay much for it. The more confident you are in *P*, however, the more you'll pay for the ticket. For example, if *P* is the proposition that a fair coin flip comes up heads, you might be willing to pay anything up to \$0.50 for the ticket. If you pay exactly \$0.50 for the ticket, then you've effectively made a bet on which you net \$0.50 if *P* is true (coin comes up heads) but lose \$0.50 if *P* is false (coin comes up tails). Seems like a fair bet.

A ticket that pays off on *P* is worth more to a rational agent the more confident she is of *P*. In fact, we typically assume that a rational agent's fair betting price for a \$1 ticket on *P* is \$cr(*P*)—she will purchase a ticket that pays \$1 on *P* for any amount *up to* cr(*P*) dollars.

For example, suppose neither you nor I know anything about the day of the week on which Frank Sinatra was born. Nevertheless, I offer to sell you the following ticket:

This ticket entitles the bearer to \$1 if Sinatra was born on a weekend, and nothing otherwise.

If you spread your credences equally among the days of the week, then \$2/7—or about \$0.29—is your fair betting price for this ticket. To buy the ticket at that price is to place a particular kind of bet that the selected day fell on a weekend. If you lose the bet, you're out the \$0.29 you paid for the ticket. If you win the bet, it cost you \$0.29 to buy a ticket which is now worth \$1, so you're up \$0.71. Why do you demand such a premium—why do you insist on a higher potential payout for this bet than the amount of your potential loss? Because you think you're more likely to lose than win, so you'll only make the bet if the (unlikely) payout is greater than the (probable) loss.

Now look at the same transaction from my point of view—the point of view of someone who's selling the ticket, and will be on the hook for \$1 if Sinatra was born on a weekend. You spread your credences equally among the days, and are willing to *buy* the ticket for *up to* \$0.29. If I spread my credences in a similar fashion, I should be willing to *sell* you this ticket for *at least* \$0.29. On the one hand, I'm handing out a ticket that may entitle you to \$1 from me once we find out about Sinatra's birthday. On the other hand, I don't think it's very likely that I'll have to pay out, so I'm willing to accept as little as \$0.29 in exchange for selling you the ticket. In general, an agent's fair betting price for a gambling ticket is both the *maximum* amount she would *pay* for that ticket and the *minimum* amount for which she would *sell* it.

All the tickets we've considered so far pay out \$1 if a particular proposition is true. Yet tickets can be bought or sold for other potential payoffs. We generally assume that the rational fair betting price for a ticket that pays \$$S$ if P is true and nothing otherwise is \$$S \cdot \text{cr}(P)$.[3] (You might think of this as the fair betting price for S tickets, each of which pays \$1 on P.) This formula works both for run-of-the-mill betting cases and for cases in which the agent has extreme opinions. For instance, consider an agent's betting behavior when her credence in P is 0. Our formula sets her fair betting price at \$0, whatever the stakes S. Since the agent doesn't think the ticket has *any* chance of paying off, she will not pay any amount of money to possess it. On the other hand, she will be willing to sell such a ticket for any amount you like, since she doesn't think she's incurring any liability in doing so.

Bayesians (and bookies) often quote bets using odds instead of fair betting prices. For instance, a bet that Sinatra was born on a weekend would typically go off at 5 to 2 odds. This means that the ratio of your potential net payout to your potential net loss is 5:2 (0.71 : 0.29). A rational agent will accept a bet on P at her odds against P (that is, cr($\sim P$) : cr(P)) or better. Yet despite the ubiquity of odds talk in professional gambling, we will use fair betting prices going forward.

9.1.1 Dutch Books for probabilism

Suppose we have an agent who violates the probability axioms by assigning both $\text{cr}(P) = 0.7$ and $\text{cr}(\sim P) = 0.7$ for some particular proposition P. (Perhaps he's a character like Mr. Bold.) Given his credence in P, this agent's fair betting price for a ticket that pays \$1 if P is true will be \$0.70. Given his credence in $\sim P$, his fair betting price for a ticket that pays \$1 if $\sim P$ is true will also be \$0.70. So let's sell him both of these tickets, at \$0.70 each.

Our agent is now in trouble. He has paid a total of \$1.40 for the two tickets, and there's no way he can make all that money back. If P is true, his first ticket is worth \$1 but his second ticket is worth nothing. If P is false, his first ticket is worth nothing and his second ticket pays only \$1. Either way, he'll be out \$0.40.

We can summarize this agent's situation with the following table:

	P	$\sim P$
Ticket pays on P	0.30	−0.70
Ticket pays on $\sim P$	−0.70	0.30
TOTAL	**−0.40**	**−0.40**

The columns of this table partition the possible states of the world. In this case, our partition contains the propositions P and $\sim P$. The agent purchases two tickets; each ticket is recorded on one row. The entries in the cells report the agent's *net* payout for that ticket in that state; all values are in dollars, and negative numbers indicate a loss. So, for instance, the upper-right cell reports that if $\sim P$ is true then the agent loses \$0.70 on his P ticket (the ticket cost him \$0.70, and doesn't win him anything in that state). The upper-left cell records that the P ticket cost the agent \$0.70, but he makes \$1 on it if P is true, for a net profit of \$0.30. The final row reports the agent's total payout for all his tickets in a given state of the world. As we can see, an agent who purchases both tickets will lose \$0.40 no matter which state the world is in. Purchasing this set of tickets guarantees him a net loss.

A **Dutch Book** is a set of bets, each placed with an agent at her fair betting price (or better), that together guarantee her a sure loss come what may.[4] The idea of a Dutch Book is much like that of a money pump (Section 7.2.1): we make a series of exchanges with the agent, each of which individually looks fair (or favorable) from her point of view, but which together yield an undesirable outcome. In a Dutch Book, each bet is placed at a price the agent considers fair given her credence in the proposition in question, but when all the bets are added up she's guaranteed to lose money no matter which possible world is actual.

Ramsey (1931, p. 84) recognized a key point about Dutch Books, which was proven by de Finetti (1937/1964):

Dutch Book Theorem: If an agent's credence distribution violates at least one of the probability axioms (Non-Negativity, Normality, or Finite Additivity), then a Dutch Book can be constructed against her.

We will prove this theorem by going through each of the axioms one at a time, and showing how to make a Dutch Book against an agent who violates it.

Non-Negativity and Normality are relatively easy. An agent who violates Non-Negativity will set a negative betting price for a ticket that pays \$1 on some proposition *P*. Since the agent assigns a negative betting price to that ticket, she is willing to sell it at a negative price. In other words, this agent is willing to pay *you* some amount of money to take a ticket which, if *P* is true, entitles you to a further \$1 from her beyond what she paid you to take it. Clearly this is a losing proposition for the agent.

Next, suppose an agent violates Normality by assigning credence greater than 1 to a tautology. Say, for instance, an agent assigns $\text{cr}(P \vee {\sim}P) = 1.4$. This agent will pay \$1.40 for a ticket that pays \$1 if $P \vee {\sim}P$ is true. The agent will definitely win that \$1, but will still have lost money overall. On the other hand, if an agent assigns a credence less than 1 to a tautology, she will sell for less than \$1 a ticket that pays \$1 if the tautology is true. The tautology will be true in every possible world, so in every world the agent will lose money on this bet.

Now let's turn to Finite Additivity. Suppose that for mutually exclusive *P* and *Q*, an agent violates Finite Additivity by assigning $\text{cr}(P) = 0.5$, $\text{cr}(Q) = 0.5$, and $\text{cr}(P \vee Q) = 0.8$. Because of these credences, the agent is willing to pay \$0.50 for a ticket that pays \$1 on *P*, and \$0.50 for a ticket that pays \$1 on *Q*. Then we have her sell us for \$0.80 a ticket that pays \$1 if $P \vee Q$.

At this point, the agent has collected \$0.80 from us and paid a total of \$1 for the two tickets she bought. So she's down \$0.20. Can she hope to make this money back? Well, the tickets she's holding will be worth \$1 if either P or Q is true. She can't win on both tickets, because P and Q were stipulated to be mutually exclusive. So at most, the agent's tickets are going to earn her \$1. But if either P or Q is true, $P \vee Q$ will be true as well, so she will have to pay out \$1 on the ticket she sold to us. The moment she earns \$1, she'll have to pay it back out. There's no way for the agent to make her money back, so no matter what happens she'll be out a net \$0.20.

The situation is summed up in this table:

	$P \,\&\, {\sim}Q$	${\sim}P \,\&\, Q$	${\sim}P \,\&\, {\sim}Q$
Ticket pays on P	0.50	−0.50	−0.50
Ticket pays on Q	−0.50	0.50	−0.50
Ticket pays on $P \vee Q$	−0.20	−0.20	0.80
TOTAL	**−0.20**	**−0.20**	**−0.20**

Since P and Q are mutually exclusive, there is no possible world in which $P \,\&\, Q$ is true, so our partition has only three members. On the first row, the P-ticket for which the agent paid \$0.50 nets her a positive \$0.50 in the state where P is true. Similarly for Q on the second row. The third row represents a ticket the agent *sold*, so she makes \$0.80 on it unless $P \vee Q$ is true, in which case she suffers a net loss. The final row sums the rows above it to show that each possible state guarantees the agent a \$0.20 loss from her bets. A similar Book can be constructed for any agent who assigns $\text{cr}(P \vee Q) < \text{cr}(P) + \text{cr}(Q)$. For a Book against agents who violate Finite Additivity by assigning $\text{cr}(P \vee Q) > \text{cr}(P) + \text{cr}(Q)$, see Exercise 9.1.

9.1.2 Further Dutch Books

We can also construct Dutch Books against agents who violate other credence requirements. For example, suppose an agent has the probabilistic unconditional credence distribution specified by the following probability table:

P	Q	cr
T	T	1/4
T	F	1/4
F	T	1/4
F	F	1/4

But now suppose that this agent violates the Ratio Formula by assigning $cr(P \mid Q) = 0.6$. To construct a Dutch Book against this agent, we first need to understand conditional bets. Suppose we sell this agent the following ticket:

If Q is true, this ticket entitles the bearer to \$1 if P is true and nothing otherwise. If Q is false, this ticket may be returned to the seller for a full refund of its purchase price.

If Q turns out to be false, the agent's full purchase price for this ticket will be refunded to her. So if Q is false, it doesn't matter what she pays; the ticket will net her \$0 no matter what. That means the agent's purchase price for this ticket should be dictated by her opinion of P in worlds where Q is true. In other words, the agent's purchase price for this ticket should be driven by $cr(P \mid Q)$. We say that this ticket creates a **conditional bet** on P given Q. A conditional bet on P given Q wins or loses money for the agent only if Q is true; if the payoff on P (given Q) is \$1, the agent's fair betting price for such a bet is $cr(P \mid Q)$. In general, conditional bets are priced using conditional credences.

Our agent who assigns $cr(P \mid Q) = 0.6$ will purchase the ticket above for \$0.60. We then have her sell us two more tickets:

1. We pay the agent \$0.25 for a ticket that pays us \$1 if $P \& Q$.
2. We pay the agent \$0.30 for a ticket that pays us \$0.60 if $\sim Q$.

Notice that Ticket 2 is for stakes other than \$1; we've calculated the agent's fair betting price for this ticket (\$0.30) by multiplying her credence in $\sim Q$ (1/2) by the ticket's payoff (\$0.60).

The agent has received \$0.55 from us, but she's also paid out \$0.60 for the conditional ticket. So she's down \$0.05. If Q is false, she'll get a refund of \$0.60 for the conditional ticket, but she'll also owe us \$0.60 on Ticket 2. If Q is true and P is true, she gets \$1 from the conditional ticket but owes us \$1 on Ticket 1. And if Q is true and P is false, she neither pays nor collects on any of the tickets and so is still out \$0.05. No matter what, the agent loses \$0.05. The following table summarizes the situation:

	$P \& Q$	$\sim P \& Q$	$\sim Q$
Ticket 1	−0.75	0.25	0.25
Ticket 2	0.30	0.30	−0.30
Conditional ticket	0.40	−0.60	0
TOTAL	**−0.05**	**−0.05**	**−0.05**

A similar Dutch Book can be constructed against any agent who violates the Ratio Formula.

David Lewis figured out how to turn this Dutch Book against Ratio Formula violators into a strategy against anyone who fails to update by Conditionalization.[5] Suppose we have an agent who assigns the unconditional credence distribution described above at t_i, satisfies the Ratio Formula, but will assign $\text{cr}_j(P) = 0.6$ if she learns Q between t_i and t_j. Since she satisfies the Ratio Formula, this agent assigns $\text{cr}_i(P \mid Q) = 0.5$. That means the $\text{cr}_j(P)$ value she will assign upon learning Q does not equal her $\text{cr}_i(P \mid Q)$ value, leaving her in violation of Conditionalization.

We take advantage of this agent's Conditionalization violation by first purchasing the Tickets 1 and 2 described above from her at t_i. The prices on these tickets match the agent's unconditional t_i credences, so she will be willing at t_i to sell them at the prices listed. We then implement the following strategy: If the agent learns Q between t_i and t_j, we sell her a ticket at t_j that pays \$1 on P. We know that this agent will assign $\text{cr}_j(P) = 0.6$ if she learns Q, so in that circumstance she'll be willing to buy this ticket for \$0.60. If the agent doesn't learn Q between t_i and t_j, we make no transactions with her beyond Tickets 1 and 2.[6]

Putting all this together, the agent's payoffs once more are:

	$P \& Q$	$\sim P \& Q$	$\sim Q$
Ticket 1	−0.75	0.25	0.25
Ticket 2	0.30	0.30	−0.30
Ticket if Q learned	0.40	−0.60	0
TOTAL	**−0.05**	**−0.05**	**−0.05**

(Because the agent purchases the third ticket only if Q is true, it neither costs nor pays her anything if Q is false.)

This agent received \$0.55 from us for selling two tickets at t_i. If Q is false, no more tickets come into play, but she owes us \$0.60 on Ticket 2, so she's out a total of \$0.05. If Q is true, she purchases the third ticket, and so is out \$0.05. If P is also true, she wins \$1 on that third ticket but has to pay us \$1 on Ticket 1, so she's still down \$0.05. If P is false (while Q is true), none of the tickets pays, and her net loss remains at \$0.05. No matter what, the agent loses money over the course of t_i to t_j.

A quick terminological remark: A Dutch Book is a specific set of bets guaranteed to generate a loss. Strictly speaking, we haven't just built a Dutch Book against Conditionalization violators, because we haven't described a single set of bets that can be placed against the agent to guarantee a sure loss in every case. Instead, we've specified *two* sets of bets, one to be placed if the agent learns Q and the other to be placed if not. (The former set contains three bets, while the latter contains two.) We've given the bookie a *strategy* for placing different sets of bets in different circumstances, such that each potential set of bets is guaranteed to generate a loss in the circumstances in which it's placed. For this reason, Lewis's argument supporting Conditionalization is usually known as a **Dutch Strategy** argument rather than a Dutch Book argument.[7]

Dutch Books or Strategies have been constructed to punish violators of many of the additional Bayesian constraints we considered in Chapter 5: the Principal Principle (Howson 1992), the Reflection Principle (van Fraassen 1984), Regularity (Kemeny 1955; Shimony 1955), Countable Additivity (Adams 1962), and Jeffrey Conditionalization (Armendt 1980; Skyrms 1987b). I will not work through the details here. Instead, we will consider the normative consequences of these Dutch constructions.

9.2 The Dutch Book Argument

A Dutch Book is a set of bets, not an argument—and neither is the Dutch Book Theorem on its own. But once we know that a Dutch Book can be constructed for a particular kind of situation, we can use that fact to argue for a norm. For example, there's the

Dutch Book Argument for Probabilism

(Premise)	No Dutch Book can be constructed against a rational agent.
(Theorem)	If an agent's credence distribution violates at least one of the probability axioms, then a Dutch Book can be constructed against her.
(Conclusion)	Every rational agent has a probabilistic credence distribution.

The key premise is that no rational agent is susceptible to being Dutch Booked; just as rational preferences help one avoid money pumps, so should rational credences save us from Dutch Books. Once we have this premise, similar Dutch Book Arguments can be constructed for the Ratio Formula, Conditionalization, and all the other norms mentioned in the previous section. But is the premise plausible? What if it turns out that a Dutch Book can be constructed against *any* agent, no matter what rules her credences do or don't satisfy? In other words, imagine that Dutch Books were just a rampant, unavoidable fact of life. That would render the Dutch Book Argument's premise false.

To reassure ourselves that we don't live in this dystopian Book-plagued world, we need a series of what Hájek (2009a) calls **Converse Dutch Book Theorems**. The usual Dutch Book Theorem tells us that if an agent violates the probability axioms, she is susceptible to a Dutch Book. A Converse Dutch Book Theorem would tell us that if an agent *satisfies* the probability axioms, she is *not* susceptible to a Dutch Book. If we had a Converse Dutch Book Theorem, then we wouldn't need to worry that whatever credences we assigned, we could be Dutch Booked. The Converse Dutch Book Theorem would guarantee us safety from Dutch bookies as long as we maintained probabilistic credences; together with the standard Dutch Book Theorem, the Converse Theorem would constitute a powerful consideration in favor of assigning probabilistic over non-probabilistic credence distributions.

Unfortunately we can't get a Converse Dutch Book Theorem of quite the kind I just described. Satisfying the probability axioms with her unconditional credences does not suffice to innoculate an agent against Dutch Books. An agent whose unconditional credence distribution satisfies the probability axioms might still violate the Ratio Formula with her conditional credences, and as we've seen this would leave her open to Book. So it can't be a theorem that no agent with probabilistic credences can ever be Dutch Booked (because that isn't true!). Instead, our Converse Dutch Book Theorem has to say that as long as an agent's credences satisfy the probability axioms, she can't be Dutch Booked *with the kind of Book we deployed against agents who violate the axioms*. For instance, if an agent satisfies Non-Negativity, there won't be any propositions to which she assigns a negative credence, so we won't be able to construct a Book against her that requires selling a ticket at a negative fair betting price (as we did against the Non-Negativity violator). Lehman (1955) and Kemeny (1955) each independently proved that if an agent's credences satisfy the probability axioms, she isn't susceptible to Books of the sort we considered in Section 9.1.1.[8] This clears the way for the Dutch Book Argument's premise to be at least plausible.

Converse Dutch Book Theorems can also help us beat back another challenge to Dutch Book Arguments. Hájek (2009a) defines a **Czech Book** as a set of bets, each placed with an agent at her fair betting price (or better), that together guarantee her a sure *gain* come what may. It's easy to see that whenever one can construct a Dutch Book against an agent, one can also construct a Czech Book. We simply take each ticket contained in the Dutch

Book, leave its fair betting price intact, but have the agent *sell* it rather than *buy* it (or vice versa). In the betting tables associated with each Book, this will flip all the negative payouts to positive and positive payouts to negative. So the total payouts on the bottom row of each column in the table will be positive, and the agent will profit come what may.

According to the Dutch Book Theorem, a Dutch Book can be constructed against any agent who violates a probability axiom. We now know that whenever a Dutch Book can be constructed, a Czech Book can be as well. This gives us the

Czech Book Theorem: If an agent's credence distribution violates at least one of the probability axioms, then a Czech Book can be constructed in her favor.

Violating the probability axioms leaves an agent susceptible to Dutch Books, which seems to be a disadvantage. But violating the probability axioms also opens up the possibility that an agent will realize Czech Books, which seems to be an advantage. Perhaps a rational agent would leave herself susceptible to Dutch Books in order to be ready for Czech Books, in which case the premise of our argument fails once more. In general, Hájek worries that any argument for probabilism based on Dutch Books will be canceled by Czech Books, leaving the Dutch Book Theorem normatively inert.

At this point, converse theorems become significant. A Converse Dutch Book Theorem says that satisfying the probability axioms protects an agent from the disadvantage of susceptibility to particular kinds of Dutch Book. But there is no Converse Czech Book Theorem—it's just not true that any agent who satisfies the probability axioms must forgo Czech Books. That's because a rational agent will purchase betting tickets at anything *up to* her fair betting price (and sell at anything *at least* that number). For instance, an agent who satisfies the axioms will assign credence 1 to any tautology and so set $1 as her fair betting price for a ticket that pays $1 if some tautology is true. But if we offer her that ticket for, say, $0.50 instead, she will be perfectly happy to take it off our hands. Since the ticket pays off in every possible world, the agent will make a profit come what may. So here we have a Czech Book available to agents who satisfy the probability axioms.

Non-probabilistic agents are susceptible to certain kinds of Dutch Books, while probabilistic agents are not. Non-probabilistic agents can take advantage of Czech Books, but probabilistic agents can too. The advantage goes to probabilism.

9.2.1 Dutch Books depragmatized

Recent authors have reformulated the Dutch Book Argument in response to two objections. First, like the Representation Theorem Argument of Chapter 8, the Dutch Book Argument seems to move from a practical premise to a theoretical conclusion. The argument establishes that an agent with non-probabilistic credences may *behave* in ways that are practically disadvantageous—buying and selling gambling tickets that together guarantee a loss. But this shows only that it's *practically* irrational to assign credences violating the probability axioms. (As with the Representation Theorem Argument, it feels like the word "practically" should be inserted before the word "rational" in both the Dutch Book Argument's premise and conclusion.) We wanted to establish the axioms as requirements of *theoretical* rationality (see Chapter 1), and the argument seems unable to do that.

The distinction between requirements of practical and theoretical rationality might disappear if one understood doxastic attitudes purely in terms of their effects on action. de Finetti, for example, explored a position that *defines* an agent's credences in terms of her betting behavior:

> Let us suppose that an individual is obliged to evaluate the rate p at which he would be ready to exchange the possession of an arbitrary sum S (positive or negative) dependent on the occurrence of a given event E, for the possession of the sum pS; we will say by definition that this number p is the measure of the degree of probability attributed by the individual considered to the event E, or, more simply, that p is the probability of E (according to the individual considered). (1937/1964, pp. 101–2)

As we discussed in Chapter 8, this kind of behaviorism about mental states is fairly unpopular these days. And even if we take this definitional approach, a second objection to the Dutch Book Argument remains: As a practical matter, susceptibility to Book doesn't seem that significant. Few of us are surrounded by bookies ready to press gambles on us should we violate the probability axioms. If the Dutch Book Argument is supposed to talk us into probabilistic credences on the grounds that failing to be probabilistic will lead to bad practical consequences, those practical consequences had better be a realistic threat.

This second objection is a bit unfair as it stands. As Julia Staffel writes:

> Dutch book arguments start from the idea that one central function of our credences is guiding our actions. Using bets as stand-ins for actions more

> generally, they are supposed to demonstrate that probabilistic credences are suitable for guiding action, whereas incoherent credences are not.... Dutch book arguments show us that coherent thinkers avoid guaranteed betting losses, hence, given that we understand bets as stand-ins for actions more generally, coherent credences are suitable for guiding our actions.
>
> (2019, pp. 57–8)

The idea is that Dutch Books aren't the only kind of practical trouble into which non-probabilistic credences might lead an agent; they're just a particularly straightforward and vivid example. While none of us may ever face a Dutch bookie, and some of us may avoiding betting entirely, there are nevertheless a variety of practical situations in which non-probabilistic credences will drive us to make sub-optimal decisions.

Still, if the concern behind Dutch Books is a practical one, shouldn't we investigate how often such situations really arise, and how much our non-probabilistic credences are liable to cost us? Maybe it's not worth the trouble to correct our degrees of belief...

To avoid these sorts of objections (and others we'll see later), recent authors have recast the Dutch Book Argument as establishing a requirement of theoretical rationality. They suggest that despite the Dutch Book's pragmatic appearance, the bookie and his Books are merely a device for dramatizing an underlying doxastic inconsistency. These authors take their inspiration from the original passage in which Ramsey mentioned Dutch Books:

> These are the laws of probability, which we have proved to be necessarily true of any consistent set of degrees of belief.... If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd. He could have a book made against him by a cunning better and would then stand to lose in any event. (1931, p. 84)

Interpreting this passage, Skyrms writes that "for Ramsey, the cunning bettor is a dramatic device and the possibility of a dutch book a striking symptom of a deeper incoherence" (1987a, p. 227). Since the bookie is only a device for revealing this deeper incoherence, it doesn't matter whether we are actually surrounded by bookies or not. (The fact that a child's current medical condition would lead her to break 100°F on a thermometer indicates an underlying problem, whether or not any thermometers are around.) As Brad Armendt puts it:

> We should resist the temptation to think that a Dutch book argument demonstrates that the violations (violations of probability, for the synchronic argument) are bound to lead to dire outcomes for the unfortunate agent. The problem is not that violators are bound to suffer, it is that their action-guiding beliefs exhibit an inconsistency. That inconsistency can be vividly depicted by imagining the betting scenario, and what would befall the violators were they in it. The idea is that the irrationality lies in the inconsistency, when it is present; the inconsistency is portrayed in a dramatic fashion when it is linked to the willing acceptance of certain loss. The value of the drama lies not in the likelihood of its being enacted, but in the fact that it is made possible by the agent's own beliefs, rather than a harsh, brutal world. (1992, p. 218)

To argue that Dutch Book vulnerability reveals a deeper rational inconsistency, we start by relating credences to betting behavior in a more nuanced manner than de Finetti's. Howson and Urbach (2006), for instance, say that an agent who assigns credence cr(P) to proposition P won't necessarily *purchase* a \$1 ticket on P at price \$cr(P), but will *regard* such a purchase as fair. (This ties the doxastic attitude of credence to another attitude—regarding as fair—rather than tying credences directly to behavior.) The Dutch Book Theorem then tells us that an agent with nonprobabilistic credences will regard each of a set of bets as fair that together guarantee a sure loss. Since such a set of bets is clearly *un*fair, a nonprobabilistic agent's degrees of belief are theoretically inconsistent because they regard as fair something that is guaranteed not to be.

Christensen (2004, Ch. 5) attenuates the connection between credences and betting rates even further. As a purely descriptive matter, an agent with particular degrees of belief may or may not regard any particular betting arrangement as fair (perhaps she makes a calculation error; perhaps she doesn't have any views about betting arrangements; etc.). Nevertheless, Christensen argues for a *normative* link between credences and fair betting prices. If an agent assigns a particular degree of belief to P, that degree of belief *sanctions as fair* purchasing a ticket for \$cr(P) that pays \$1 on P; it *justifies* the agent's evaluating such a purchase as fair; and it makes it *rational* for the agent to purchase such a ticket at (up to) that price.

Christensen then argues to probabilism from three premises:[9]

<u>Depragmatized Dutch Book Argument for Probabilism</u>

(Premise) An agent's degrees of belief sanction as fair monetary bets at odds matching her degrees of belief. (Christensen calls this premise "Sanctioning".)

(Premise) A set of bets that is logically guaranteed to leave an agent monetarily worse off is rationally defective. ("Bet Defectiveness")

(Premise) If an agent's beliefs sanction as fair each of a set of bets, and that set of bets is rationally defective, then the agent's beliefs are rationally defective. ("Belief Defectiveness")

(Theorem) If an agent's degrees of belief violate the probability axioms, there exists a set of bets at odds matching her degrees of belief that is logically guaranteed to leave her monetarily worse off.

(Conclusion) If an agent's degrees of belief violate the probability axioms, that agent's degrees of belief are rationally defective.

The theorem in this argument is, once more, the Dutch Book Theorem, and the argument's conclusion is a version of probabilism. Christensen assesses this kind of Dutch Book Argument ("DBA") as follows:

> This distinctively non-pragmatic version of the DBA allows us to see why its force does not depend on the real possibility of being duped by clever bookies. It does not aim at showing that probabilistically incoherent degrees of belief are unwise to harbor for practical reasons. Nor does it locate the problem with probabilistically incoherent beliefs in some sort of preference inconsistency. Thus it does not need to identify, or define, degrees of belief by the ideally associated bet evaluations. Instead, this DBA aims to show that probabilistically incoherent beliefs are rationally defective by showing that, in certain particularly revealing circumstances, they would provide *justification* for bets that are rationally defective in a particularly obvious way. The fact that the diagnosis can be made *a priori* indicates that the defect is not one of fitting the beliefs with the way the world happens to be: it is a defect internal to the agent's belief system. (2004, p. 121, emphasis in original)

9.3 Objections to Dutch Book Arguments

If we can construct both a Dutch Book Theorem and a Converse Dutch Book Theorem for a particular norm (probabilism, the Ratio Formula, updating by Conditionalization, etc.), then we have a Dutch Book Argument that rationality requires honoring that norm. I now want to review various objections to Dutch Book Arguments that have arisen over the years; these objections apply equally well to depragmatized versions of such arguments.

It's worth beginning with a concern that is often overlooked. Dutch Book Arguments assume that a rational agent's fair betting price for a bet that pays \$1 on P is \$cr(P). An author like de Finetti who *identifies* an agent's credence in P with the amount she's willing to pay for a ticket that yields \$1 on P is free to make this move. But contemporary authors unwilling to grant that identification need some argument that these betting prices are rationally required.

A simple argument comes from expected value calculations. As we saw in Chapter 7, Equation (7.3), an agent's expected monetary payout for a ticket that pays \$1 on P is

$$\$1 \cdot \mathrm{cr}(P) + \$0 \cdot \mathrm{cr}(\sim P) = \$\mathrm{cr}(P) \tag{9.1}$$

So an agent whose preferences are driven by expected value calculations will assign that ticket a fair betting price of \$cr(P). (The calculation can be generalized to bets at other stakes.)

But this argument for the fair betting prices we've been assuming takes as a premise that rational agents maximize expected utility. (Or expected monetary return—recall that we assumed for the duration of this chapter that agents assign constant marginal utility to money.) If we had *that* premise available, we could argue much more directly for probabilism via the Revised Representation Theorem of Chapter 8.[10]

So what else can we try? At the beginning of Section 9.1, I tried to motivate the typical formula for fair betting prices without appealing to expectations. I invoked intuitions about how an agent's fair betting price for a ticket should rise and fall as her credences and the stakes change. Unfortunately, those intuitive motivations can't take us quite far enough. An agent could assign fair betting prices that rise and fall in the manner described without setting those fair betting prices equal to her credences.

Recall Mr. Bold, who assigns to each proposition the square-root of the credence assigned by Mr. Prob. Mr. Prob's credences satisfy the probability axioms, while Mr. Bold's violate Finite Additivity. Now suppose that Mr. Bold sets his fair betting prices for various gambling tickets equal not to his credences, but instead to the *square* of his credences. Mr. Bold's fair betting prices (for tickets on contingent propositions) will still rise and fall in exactly the ways that intuition requires. In fact, he will be willing to buy or sell any gambling ticket at exactly the same prices as Mr. Prob. And since Mr. Prob isn't susceptible to various kinds of Dutch Book, Mr. Bold won't be either. In general, an agent who assigns nonprobabilistic credences may be able to avoid

Book by assigning his betting prices in nonstandard fashion. Without a strong assumption about how rational agents use their credences to set betting prices, the Dutch Book Argument cannot show that nonprobabilistic credences are irrational.[11]

9.3.1 The Package Principle

The objection just raised applies to any Dutch Book Argument, because it questions how fair betting prices are set for the bets within a Book. Another, more traditional objection applies only to Books involving more than one gambling ticket; for instance, it applies to the Dutch Book against Finite Additivity violators but not to the Books against Non-Negativity and Normality offenders. (As I keep saying, Finite Additivity is the most difficult of the three axioms to establish as a rational rule.)

This traditional objection begins with **interference effects** that may be generated by placing a series of bets in succession. Interference effects occur when the initial bets in a series interfere with an agent's willingness to accept the remaining bets. While she might have accepted the remaining bets as fair had they been offered to her in isolation, the bets she's made already turn her against them. For example, the agent might have a personal policy never to tie up more than a certain total amount of money in gambles at one time. Or the third bet she's offered might be on the proposition "I will never make more than two bets in my life." More to the point, suppose we have an agent whose credences violate the probability axioms; we carefully construct a set of bets guaranteeing a sure loss, each of which will be placed at odds matching her degree of belief in the relevant proposition. We offer these bets to her one at a time. There's no guarantee that placing the first few wagers won't interfere with the agent's willingness to accept the remainder. Besides the interference effects just mentioned, the agent might see her sure loss coming down the pike, and simply refuse to place any more bets past some point! Interference effects undermine the claim that any agent with nonprobabilistic credences can be trapped into placing a sure-loss set of bets.

Interference effects are often introduced (as I've just done) by talking about placing bets with an agent one at a time. A Dutch Book defender might respond by suggesting that the bookie place his bets with the agent all at once—as a package deal. Yet the agent might still reject this package on the grounds that she doesn't like to tie up so much money in gambles, or that she can see a sure loss on the way. The sequential offering of the bets over time is ultimately irrelevant to the dialectic. A more promising response to interference effects points out how heavily they rely on the transactional pragmatics of betting. Depragmatized Dutch Book Arguments indict a nonprobabilistic agent on the grounds that her credences *sanction* a sure-loss set of bets; whether interference effects would impede her actually *placing* those bets is neither here nor there.

Yet there's a problem in the vicinity even for depragmatized arguments. Howson and Urbach's and Christensen's arguments contend that Dutch Bookability reveals an underlying doxastic inconsistency. What's the *nature* of that inconsistency? Earlier we saw Ramsey suggesting of the probability rules that, "If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd." From this suggestion, Skyrms takes the principle that for a rational agent, "A betting arrangement gets the same expected utility no matter how described" (1987a, p. 230). Similarly, Joyce writes that an agent's nonprobabilism "leads her to commit both the prudential sin of squandering happiness and the epistemic sin of valuing prospects differently depending on how they happen to be described"(1998, p. 96).

The rational inconsistency revealed by Dutch Bookability seems to be that the agent evaluates *one and the same entity* differently depending on how it is presented. Skyrms calls the entity being evaluated a "betting arrangement". To illustrate how one and the same betting arrangement might be presented in two different ways, let's return to our Dutch Book against a Finite Additivity violator (Section 9.1.1). That Book begins by selling the agent a ticket that pays \$1 if P is true and another ticket that pays \$1 on Q (call these the "P-ticket" and the "Q-ticket", respectively). The agent assigns $\text{cr}(P) = \text{cr}(Q) = 0.5$, so she will buy these tickets for \$0.50 each. At that point the agent has purchased a package consisting of two tickets:

This ticket entitles the bearer to \$1 if P is true, and nothing otherwise.	This ticket entitles the bearer to \$1 if Q is true, and nothing otherwise.

Call this the "PQ-package". We assume that since the agent is willing to pay \$0.50 for each of the two tickets in the package, she will pay \$1 for the package as a whole.

In the next step of the Finite Additivity Dutch Book, we buy the following ticket from the agent (which we'll call the "$P \vee Q$-ticket"):

This ticket entitles the bearer to \$1 if $P \vee Q$ is true, and nothing otherwise.

Our agent assigns $\mathrm{cr}(P \vee Q) = 0.8$, so she sells us this ticket for \$0.80.

Now compare the *PQ*-package with the $P \vee Q$-ticket, and keep in mind that in this example *P* and *Q* are mutually exclusive. If either *P* or *Q* turns out to be true, the *PQ*-package and the $P \vee Q$-ticket will each pay exactly \$1. Simlarly, each one pays \$0 if neither *P* nor *Q* is true. So the *PQ*-package pays out the same amount as the $P \vee Q$-ticket in every possible world. This is the sense in which they represent the same "betting arrangement". When we offer the agent that betting arrangement as a package of two bets on atomic propositions, she values the arrangement at \$1. When we offer that arrangement as a single bet on a disjunction, she values it at \$0.80. She values the same thing—the same betting arrangement—differently under these two presentations. If she's willing to place bets based on those evaluations, we can use them to take money from her. (We sell the arrangement to her in the form she holds dear, then buy it back in the form she'll part with for cheap.) But even if the agent won't actually place the bets, the discrepancy in her evaluations reveals a rational flaw in her underlying credences.

The general idea is that any Dutch Book containing multiple bets reveals a violation of

Extensional Equivalence: If two betting arrangements have the same payoff as one another in each possible world, a rational agent will value them equally.

I certainly won't question Extensional Equivalence. But the argument above sneaks in another assumption as well. How did we decide that our agent valued the *PQ*-package at \$1? We assumed that since she was willing to pay \$0.50 for the *P*-ticket on its own and \$0.50 for the *Q*-ticket as well, she'd pay \$1 for these two tickets bundled together as a package. We assumed the

Package Principle: A rational agent's value for a package of bets equals the sum of her values for the individual bets it contains.

Our argument needed the Package Principle to get going (as does every Dutch Book consisting of more than one bet). We wanted to indict the set of credences our agent assigns to *P*, *Q*, and $P \vee Q$. But bets based on those individual propositions would not invoke Extensional Equivalence, because no two such bets have identical payoffs in each possible world. So we *combined* the *P*- and *Q*-tickets into the *PQ*-package, a betting arrangement extensionally equivalent to the $P \vee Q$-ticket. We then needed a value for the *PQ*-package, a new object not immediately tied to any of the agent's credences. So we applied the Package Principle.[12]

Is it legitimate to assume the Package Principle in arguing for Finite Additivity? I worry that we face a Linearity In, Linearity Out problem once more. In order to get a Dutch Book Argument for Finite Additivity, we need to assume that a rational agent's value for a package of bets on mutually exclusive propositions is a linear combination of her values for bets on the individual propositions. Schick (1986, p. 113) calls this "the unspoken assumption . . . of value additivity" in Dutch Book Arguments; it seems to do exactly for bet valuations what Finite Additivity does for credences.[13] Without an independent argument for this Package Principle, the Dutch Book Argument for probabilism cannot succeed.[14]

9.3.2 Dutch Strategy objections

The first objection discussed in this section—concerning fair betting prices—applies to any Dutch Book. The Package Principle objection applies to Books containing multiple bets. But even beyond those objections, special problems arise for Dutch arguments involving credences assigned at different times. I will focus here on Lewis's Dutch Strategy Argument for Conditionalization; similar points apply to Strategies supporting Jeffrey Conditionalization and other potential diachronic norms.

To get the concern going, we need to focus on an aspect of Dutch Books and Strategies that I haven't mentioned yet. I keep saying that a Dutch Book guarantees the agent will lose money in every possible world. What set of possible worlds am I talking about? It can't be the set of logically possible worlds; after all, there are logically possible worlds in which no bets are ever placed. When we say that a Dutch Book guarantees the agent a sure loss in every world, we usually mean something like the agent's doxastically possible worlds—the worlds she entertains as a live option.

It makes sense to construct Dutch Books around worlds the agent considers possible. Dutch Book susceptibility is supposed to be a *rational* flaw, and rationality concerns how things look from the agent's own point of view. Imagine a bookie sells you for \$0.50 a bet that pays \$1 if a particular fair coin flip comes up heads. The bookie then claims he has Dutch Booked you, because he's already seen the flip outcome and it came up tails! Your willingness to purchase that bet didn't reveal any rational flaw in your credences. Admittedly there's *some* sense in which the bookie sold you a bet that's a loser in every live possibility. But that's a sense of "live possibility" to which you didn't have access when you placed the bet; relative to *your* information the bet wasn't a sure loss. To constrain our attention to Dutch Books or Strategies capable of revealing *rational* flaws, we usually require them to generate a sure loss across the agent's space of doxastically possible worlds. A convenient way to do this is to stipulate that the bookie in a Dutch Book or Strategy must be capable of constructing the Book or implementing the Strategy without employing any contingent information the agent lacks.

With that in mind, let's return to Lewis's Dutch Strategy against Conditionalization violators. Here's the particular set of bets we used in Section 9.1.2:

	$P \& Q$	$\sim P \& Q$	$\sim Q$
Ticket 1	−0.75	0.25	0.25
Ticket 2	0.30	0.30	−0.30
Ticket if Q learned	0.40	−0.60	0
TOTAL	**−0.05**	**−0.05**	**−0.05**

This Strategy was constructed against an agent who assigns equal unconditional credence to each of the four P/Q state-descriptions at t_i, assigns $\text{cr}_i(P \mid Q) = 0.5$, yet assigns $\text{cr}_j(P) = 0.6$ if she learns that Q between t_i and t_j.

At a first pass, this Strategy seems to meet our requirement that the bookie need not know more than the agent. Tickets 1 and 2 are purchased from the agent at t_i using betting prices set by her t_i credences. The third ticket is sold to the agent at t_j only if she learns Q between t_i and t_j. But by t_j the agent (and the bookie) know whether she has learned Q, so the bookie needn't know more than the agent to decide whether to sell that ticket.

Yet matters turn out to be more subtle than that. To see why, I'd suggest that the reader construct a Dutch Strategy against an agent who assigns the same t_i credences as in our example but assigns $\text{cr}_j(P) = 0.4$ if she learns that Q between t_i and t_j. Obviously the change in $\text{cr}_j(P)$ value changes the bet made if Q is learned; that bet must be keyed to the agent's fair betting prices at the later time. More interestingly, though, you'll find that while the bets placed at t_i have much the same structure as Ticket 1 and Ticket 2, the bookie needs to *sell* them at t_i—rather than *buying* them—in order to guarantee the agent a sure loss.

Now imagine a bookie is confronted at t_i by an agent who assigns equal credence to all four P/Q state-descriptions and satisfies the probability axioms and Ratio Formula. The bookie wants to initiate a Dutch Strategy that will cost the agent money should she fail to update by Conditionalization at t_j. But the bookie doesn't know which Strategy to pursue: the Strategy against agents who assign $\text{cr}_j(P) > 0.5$, or the Strategy against $\text{cr}_j(P) < 0.5$. These Strategies require the bookie to take different sides on his t_i bets, so in order to pursue a course that will definitely cost the agent, the bookie must know at t_i what the agent's credences will be at t_j. Given our stipulation that the bookie knows only what the agent does, this means that a Dutch Strategy can be constructed against an agent who violates Conditionalization only if that agent knows in advance how she'll be violating it.

How might an agent know in advance what credences she'll assign in the future? One possibility is if the agent has a standing policy, or plan, for updating in response to evidence. If the agent has such a plan, and it recommends different credences than Conditionalization, then the agent (and bookie) will be able to tell at t_i that she'll violate Conditionalization at t_j. Moreover, if the agent knows at t_i that, say, her updating plan would lead her to assign $\text{cr}_j(P) = 0.6$ upon learning that Q, the bookie can take advantage of this information to set up a Dutch Strategy by placing the appropriate bets at t_i.

By itself, violating Conditionalization doesn't make an agent susceptible to a Dutch Strategy. A Strategy can be implemented against an agent only if she violates Conditionalization *after planning to do so*. To dramatize the point, consider an agent who plans at t_i to conditionalize, but when t_j comes around actually violates Conditionalization. No Dutch Strategy can be implemented against such an agent; since at t_i the bookie won't know the details of the violation, he won't be able to place the requisite t_i bets.

Once we see this point, we might wonder whether the rational fault lies in the plan, or in the implementation. van Fraassen writes:

> Let us emphasize especially that these features are demonstrable *beforehand*, without appeal to any but logical considerations, and the strategy's implementation requires no information inaccessible to the agent himself. The general conclusion must be that an agent vulnerable to such a Dutch strategy has an initial state of opinion and practice of changing his opinion, which together constitute a demonstrably bad guide to life.
>
> (1984, p. 240, emphasis in original)

According to van Fraassen, it's the initial state of opinion plus the *initial practice of changing opinion* that together constitute a bad guide. If that's right, then the

Dutch Strategy targets a synchronic inconsistency among stances adopted at t_i, not any truly diachronic inconsistency.

One might respond that it's irrational for an agent to make one plan then carry out another, so a rational agent plans to conditionalize just in case she does so. (If that's right, then any argument that a rational agent plans to conditionalize is also an argument that she does conditionalize.) Notice first that this biconditional deems it irrational to update without first planning to do so—must rationality bar spontaneity? But setting that point aside, there's a legitimate question about how to *argue* for the diachronic rational requirement that an agent update as she had planned. For the previous few paragraphs, I've been trying to establish that this requirement cannot be established using a Dutch Strategy. But if you still aren't convinced, the following analogy may help.

Suppose my sister and I each have credences satisfying the probability axioms and Ratio Formula, but I assign $\text{cr}(P) = 0.7$, while she assigns $\text{cr}(\sim P) = 0.7$. A clever bookie could place bets with each of us that together guaranteed him a sure profit. Should this bother me? Or my sister? Not unless we antecedently think there's something wrong with our having differing opinions.

Now consider an agent who, at t_j, takes her credences to stand in the same relation to her t_i assignments that I take my credences to stand in to my sister's. This t_j agent doesn't see anything rationally pressing about lining up her credences with the credences or plans she made at t_i. A bookie might place a series of bets with the agent's t_i and t_j selves that together guarantee a sure loss. But the t_j agent will find that no more impressive than the sure-loss contract constructible against me and my sister. If the t_j agent doesn't antecedently feel any rational pressure to coordinate her current attitudes with those of her t_i self, pointing out that their combined activities result in a guaranteed loss will not create that pressure.

A Dutch Strategy may establish that it's rational for an agent to *plan* to Conditionalize. But it cannot establish that an agent is rationally required at a later time to do what she planned earlier. There may of course be *other* arguments for such a diachronic rational requirement, but they must be independently established before a Dutch Strategy can have any diachronic bite. As Christensen puts it:

> Without some independent reason for thinking that an agent's present beliefs must cohere with her future beliefs, her potential vulnerability to the Dutch strategy provides no support at all for [conditionalization]. (1991, p. 246)

A Dutch Strategy Argument may fill out the details of rational updating norms should any exist. But it is ill-suited to establish the existence of such norms in the first place.[15]

9.4 Exercises

Problem 9.1. 🌶 In Section 9.1.1 we constructed a Dutch Book against an agent like Mr. Bold whose credences are subadditive. Now construct a Dutch Book against an agent whose credences are *super*additive: for mutually exclusive P and Q, he assigns $\text{cr}(P) = 0.3$, $\text{cr}(Q) = 0.3$, but $\text{cr}(P \vee Q) = 0.8$. Describe the bets composing your Book, say why the agent will find each one acceptable, and show that the bets guarantee him a loss in every possible world.

Problem 9.2. 🌶🌶 Roxanne's credence distribution at a particular time includes the following values:

$$\text{cr}(A \,\&\, B) = 0.5 \qquad \text{cr}(A) = 0.1 \qquad \text{cr}(B) = 0.5 \qquad \text{cr}(A \vee B) = 0.8$$

(Do *not* assume that A and B are mutually exclusive!)

(a) Show that Roxanne's distribution violates the probability axioms.

(b) Construct a Dutch Book against Roxanne's credences. Lay out the bets involved, then show that those bets actually constitute a Dutch Book against Roxanne.
Note: The Book must be constructed using only the credences described above; since Roxanne is non-probabilistic you may not assume anything about the other credences she assigns. However, the Book need not take advantage of all four credences.

(c) Construct a Czech Book in Roxanne's favor. Lay out the bets involved and show that they guarantee her a profit in every possible world.

(d) Does the success of your Dutch Book against Roxanne require her to satisfy the Package Principle? Explain.

Problem 9.3. 🌶🌶 You are currently certain that you are not the best singer in the world. You also currently satisfy the probability axioms and the Ratio Formula. Yet you assign credence 0.5 that you will go to a karaoke bar tonight, and are certain that if you *do* go to the bar, cheap beer and persuasive friends will make you certain that you *are* the best singer in the world. Suppose a bookie offers you the following two betting tickets right now:

This ticket entitles you to $20 if you go to the bar, and nothing otherwise.

If you go to the bar, this ticket entitles you to $40 if you are not the world's best singer, and nothing if you are. If you don't go to the bar, this ticket may be returned to the seller for a full refund of its purchase price.

(a) Suppose that right now, a bookie offers to sell you the first ticket above for $10 and the second ticket for $30. Explain why, given your current credences, you will be willing to buy the two tickets at those prices. (Remember that the second ticket involves a conditional bet, so its fair betting price is determined by your current *conditional* credences.)
(b) Describe a Dutch Strategy the bookie can plan against you right now. In particular, describe a third bet that he can plan to place with you later tonight only if you're at the bar and certain of your singing prowess, such that he's guaranteed to make a net profit from you come what may. Be sure to explain why you'll be willing to accept that third bet later on, and how it creates a Dutch Strategy against you.[16]

Problem 9.4. ✎ Do you think there is any kind of Dutch Book or Strategy that reveals a rational flaw in an agent's attitudes? If so, say why, and say which *kinds* of Dutch Books/Strategies you take to be revealing. If not, explain why not.

9.5 Further reading

INTRODUCTIONS AND OVERVIEWS

Susan Vineberg (2011). Dutch Book Arguments. In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Summer 2011

Covers all the topics discussed in this chapter in much greater depth, with extensive citations.

CLASSIC TEXTS

Frank P. Ramsey (1931). Truth and Probability. In: *The Foundations of Mathematics and Other Logic Essays*. Ed. by R. B. Braithwaite. New York: Harcourt, Brace and Company, pp. 156–98

Bruno de Finetti (1937/1964). Foresight: Its Logical Laws, its Subjective Sources. In: *Studies in Subjective Probability*. Ed. by Henry E. Kyburg Jr and H.E. Smokler. New York: Wiley, pp. 94–158. Originally published as "La prévision; ses lois logiques, ses sources subjectives" in *Annales de l'Institut Henri Poincaré*, Volume 7, 1–68

On p. 84, Ramsey notes that any agent whose degrees of belief violated the laws of probability "could have a book made against him by a cunning better and would then stand to lose in any event." de Finetti goes on to prove it.

Frederic Schick (1986). Dutch Bookies and Money Pumps. *The Journal of Philosophy* 83, pp. 112–19

Compares Dutch Book and money pump arguments, then offers a Package Principle objection to each.

Paul Teller (1973). Conditionalization and Observation. *Synthese* 26, pp. 218–58

First presentation of Lewis's Dutch Strategy Argument for Conditionalization; also contains a number of other interesting arguments for Conditionalization.

EXTENDED DISCUSSION

David Christensen (2001). Preference-based Arguments for Probabilism. *Philosophy of Science* 68, pp. 356–76

Presents depragmatized versions of both the Representation Theorem and Dutch Book Arguments for probabilism, then responds to objections.

Notes

1. Or something very close to Kolmogorov's rules—the agent may assign a maximal credence other than 1.
2. Over the years, Bayesians have used the term **coherent** in a variety of ways. de Finetti defined a "coherent" credence distribution as one that avoided Dutch Book, then proved

that coherence requires probabilistic credences. Other Bayesian authors take satisfying the probability axioms as the definition of "coherence". Some use "coherent" simply as a synonym for "rationally consistent", whatever such consistency turns out to require.

3. This formula is easy to derive if we assume that the agent selects her fair betting prices so as to maximize expected dollar return, as we did in Section 7.1. Yet I've been scrupulously avoiding making that assumption here, for reasons I'll explain in Section 9.3.
4. A "book" is a common term for a bet placed with a "bookmaker" (or "bookie"), but why *Dutch*? Hacking (2001, p. 169) attributes the term to Ramsey, and suggests it may have been English betting slang of Ramsey's day. Concerned to avoid using what is possibly a slur, Hacking prefers to speak of "sure-loss contracts". Yet I haven't been able to find the "Dutch" terminology anywhere in Ramsey's text, so its origins remain a mystery to me. Since the phrase "Dutch Book" is near-ubiquitous in the Bayesian literature, and we have no evidence that (or how) it gives offence, I will continue to use it here. But I'd be happy for Bayesians to move in another direction if more information came to light.
5. Lewis didn't publish this argument against non-Conditionalizers. Instead it was reported by Teller (1973), who attributed the innovation to Lewis.
6. Notice that the ticket sold to the agent at t_j does not constitute a conditional bet. It's a normal ticket paying out $1 on *P* no matter what, and we set the agent's fair betting price for this ticket using her unconditional credences at t_j. It's just that we decide whether to sell her this (normal, unconditional) ticket on the basis of what she learns between t_i and t_j.

   Notice also that for purposes of this example we're assuming that the agent learns *Q* between t_i and t_j just in case *Q* is true.
7. Because I like to keep things tidy, I have constructed all of my Dutch Books and Dutch Strategies above so that the agent loses the same amount in each possible world. That isn't a requirement for Dutch Books or Strategies, and you certainly don't have to follow it when constructing your own—as long as the net payout in each world is negative, the goal has been achieved.
8. The phrase "Books of the sort we considered in Section 9.1.1" underspecifies the precise sets of Books in question; Lehman and Kemeny are much clearer about what kinds of betting packages their results concern. See also an early, limited Converse result in (de Finetti 1937/1964, p. 104).
9. I have slightly altered Christensen's premises to remove his references to a "simple agent". Christensen uses the simple agent to avoid worries about the declining marginal utility of money and about the way winning one bet may alter the value of a second's payout. Inserting the simple agent references back into the argument would not protect it from the objections I raise in the next section.
10. For more on the question of whether rationality requires agents to set their fair betting prices using expected utilities, see Hedden (2013), Wroński and Godziszewski (2017), and Pettigrew (2021).
11. All this should sound very reminiscent of my criticisms of Representation Theorem Arguments in Chapter 8. Mathematically, the proof of the Revised Representation Theorem from that chapter is very similar to standard proofs of the Dutch Book Theorem.

12. A similar move is hidden in Christensen's "Belief Defectiveness" principle. The principle says that if an agent's degrees of belief sanction as fair each bet in a set of bets, and that set of bets is rationally defective, then the agent's beliefs are rationally defective. The intuitive idea is that beliefs which sanction something defective are themselves defective. Yet without the Package Principle, an agent's beliefs might sanction as fair each of the bets in a particular set without sanctioning the *entire set* as fair. And it's the entire set of bets that is the rationally defective object—it's the *set* that guarantees the agent a sure loss.
13. Schick contends that money pump arguments for the Preference Axioms (Section 7.2.1) also assume something like the Package Principle.
14. If we assume that rational agents maximize expected utility, we can generate straightforward arguments for both Extensional Equivalence and the Package Principle. But again, if we are allowed to make *that* assumption, then probabilism follows quickly. (In fact, Extensional Equivalence is a key lemma in proving the Revised Representation Theorem.)
15. For what it's worth, one could make a similar point about Dutch Book Arguments for *synchronic* norms relating distinct degrees of belief. To dramatize the point, imagine that an agent's propositional attitudes were in fact little homunculi, each assigned its own proposition to tend to and adopt a degree of belief toward. If we demonstrated to one such homunculus that combining his assignments with those of other homunculi would generate a sure loss, he might very well not care.

    The point of this fanciful scenario is that while Dutch Books may fill out the details of rational relations among an agent's degrees of belief at a given time, they are ill-suited to establish that rationality requires such synchronic relations in the first place. Absent an antecedent rational pressure to coordinate attitudes adopted at the same time, the fact that such attitudes could be combined into a sure loss would be of little normative significance. No one ever comments on this point about Dutch Books, because we all assume in the background that degrees of belief assigned by the same agent at the same time are required to stand in some relations of rational coherence.
16. I owe this entire problem to Sarah Moss.

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