Fundamentals of Bayesian Epistemology 1

Introducing Credences

MICHAEL G. TITELBAUM

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Great Clarendon Street, Oxford, OX2 6DP,
United Kingdom

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide. Oxford is a registered trade mark of
Oxford University Press in the UK and in certain other countries

First Edition published in 2022
Impression: 1

Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2021949533

ISBN 978-0-19-870760-8 (hbk.)
ISBN 978-0-19-870761-5 (pbk.)

DOI: 10.1093/oso/9780198707608.001.0001

Printed and bound by
CPI Group (UK) Ltd, Croydon, CR0 4YY

Contents

VOLUME 2

III. APPLICATIONS

IV. ARGUMENTS FOR BAYESIANISM

5
Further Rational Constraints

The previous three chapters have discussed five core normative Bayesian rules: Kolmogorov's three probability axioms, the Ratio Formula, and Conditionalization. Bayesians offer these rules as necessary conditions for an agent's credences to be rational. We have not discussed whether these five rules are jointly sufficient for rational credence.

Agents can satisfy the core rules and still have wildly divergent attitudes. Suppose 1,000 balls have been drawn from an urn and every one of them has been black. In light of this evidence, I might be highly confident that the next ball drawn will be black. But I might also have a friend Mina, whose credences satisfy all the rational constraints we have considered so far, yet who nevertheless responds to the same evidence by being 99% confident that the next ball will be white. Similarly, if you tell me you rolled a fair die but don't say how the roll came out, I might assign credence 1/6 that it came up six. Mina, however, could be 5/6 confident of that proposition, without violating the core Bayesian rules in any way.

If we think Mina's credences in these examples are irrational, we need to identify additional rational requirements beyond the Bayesian core that rule them out. We have already seen one potential requirement that goes beyond the core: the Regularity Principle (Section 4.2) prohibits assigning credence 0 to logically contingent propositions. What other requirements on rational credence might there be? When all the requirements are put together, are they strong enough to dictate a single rationally permissible credence distribution for each possible body of total evidence?

The answer to this last question is sometimes taken to separate Subjective from Objective Bayesians. Unfortunately, "Objective/Subjective Bayesian" terminology is used ambiguously, so this chapter begins by clarifying two different ways in which those terms are used. In the course of doing so we'll discuss various interpretations of probability, including frequency and propensity views.

Then we will consider a number of additional rational credence constraints proposed in the Bayesian literature. We'll begin with synchronic constraints: the Principal Principle (relating credences to chances); the Reflection Principle

Fundamentals of Bayesian Epistemology 1: Introducing Credences. Michael G. Titelbaum, Oxford University Press.
 DOI: 10.1093/oso/9780198707608.003.0005

(concerning one's current credences about one's future credences); principles for deferring to experts; indifference principles (for distributing credences in the absence of evidence); and principles for distributing credences over infinitely many possibilities. Finally, we will turn to Jeffrey Conditionalization, a diachronic updating principle proposed as a generalization of standard Conditionalization.

Most of these constraints are usually offered as *supplements* to the five core Bayesian rules we've seen already. You may not have noticed, but in discussing Conditionalization and drawing out its consequences, we assumed throughout that the updating agents satisfied the probability axioms and Ratio Formula. Similarly, most of the principles we will discuss in this chapter build upon the five core rules, and only have their intended effects if those five rules are assumed in the background. Jeffrey Conditionalization, on the other hand, is sometimes proposed as a *substitute* for Conditionalization—though it still assumes the other four, synchronic core rules.

5.1 Subjective and Objective Bayesianism

When a weather forecaster comes on television and says, "The probability of snow tomorrow is 30%," what does that mean? What exactly has this forecaster communicated to her audience? Such questions have been asked throughout the history of mathematical probability theory; in the twentieth century, rival answers became known as **intepretations of probability**. There is an excellent literature devoted to this topic and its history (see the Further Reading of this chapter for recommendations), so I don't intend to let it take over this book. But for our purposes it's important to touch on some of the main interpretations, and at least mention some of their advantages and disadvantages.

5.1.1 Frequencies and propensities

The earliest European practitioners of mathematical probability theory applied what we now call the **classical interpretation** of probability. This interpretation, championed most famously by Pierre-Simon Laplace, calculates the probability of a proposition by counting up the number of possible event outcomes consistent with that proposition, then dividing by the total number of outcomes possible. For example, if I roll a six-sided die, there are six possible outcomes, and three of them are consistent with the proposition that the die came up even. So the classical probability of even is 1/2. (This is almost certainly the kind of probability you first encountered in school.)

Laplace advocated this procedure for any situation in which "nothing leads us to believe that one of [the outcomes] will happen rather than the others" (Laplace 1814/1995, p. 3). Applying what Jacob Bernoulli (1713) had earlier called the "principle of insufficient reason", Laplace declared that in such cases we should view the outcomes as "equally possible", and calculate the probabilities as described above.

The notion of "equally possible" at the crux of this approach clearly needs more philosophical elucidation. But even setting that aside, the classical interpretation leaves us adrift the moment someone learns to shave a die. With the shape of the die changed, our interpretation of probability needs to allow the possibility that some faces are more probable than others. For instance, it might now be 20% probable that you will roll a six. While Laplace recognized and discussed such cases, it's unclear how his view can interpret the probabilities involved. There are no longer possible outcomes of the roll that can be tallied up and put into a ratio equaling 20%.

So suppose a shady confederate offers to sell you a shaved die with "a 20% probability of landing six". How might she explain—or back up—that probability claim? Well, if an event has a 20% probability of producing a certain outcome, we expect that were the event repeated it would produce that type of outcome roughly 20% of the time. The **frequency theory** of probability uses this fact to analyze "probability" talk. On this interpretation, when your confederate claims the die has a 20% probability of landing six on a given roll, she *means* that repeated rolls of the die will produce a six about 20% of the time. According to the frequency theory, the probability is x that event A will have outcome B just in case proportion x of events like A have outcomes like B.[1] The frequency theory originated in work by Robert Leslie Ellis (1849) and John Venn (1866), then was famously developed by the logical positivist Richard von Mises (1928/1957).

The frequency theory has a number of problems; I will mention only a few.[2] Suppose my sixteen-year-old daughter asks for the keys to my car; I wonder what the probability is that she will get into an accident should I give her the keys. According to the frequency theory, the probability that the event of my giving her the keys will have the outcome of an accident is determined by how frequently this type of event leads to accidents. But what type of event is it? Is the frequency in question how often people who go driving get into accidents? How often sixteen-year-olds get into accidents? How often sixteen-year-olds with the courage to ask their fathers for the keys get into accidents? How often my daughter gets into accidents? Presumably these frequencies will differ—which one is *the* probability of an accident should I give my daughter the keys right now?

Any event can be subsumed under many types, and the frequency theory leaves it unclear which event-types determine probability values. Event types are sometimes known as reference classes, so this is the **reference class problem**. In response, one might suggest that outcomes have frequencies—and therefore probabilities—only *relative* to the specification of a particular reference class (either implicitly or explicitly). But it seems we can meaningfully inquire about the probabilities of particular event outcomes (or of propositions simpliciter) without specifying a reference class. I need to decide whether to give the keys to my daughter; I want to know how probable it is that she will crash. That probability doesn't seem relative to any particular reference class. Or if it is (covertly) relative to some reference class, which reference class does the job?

Frequency information about specific event-types seems more relevant to determining probabilities than information about general types. (The probability that my daughter will get into an accident on this occasion seems much closer to *her* frequency of accidents than to the accident frequency of drivers in general.) Perhaps probabilities are frequencies in the maximally specific reference class? But the *maximally* specific reference class containing a particular event contains only that individual event. The frequency with which my daughter gets into an accident when I give her my keys *on this occasion* is either 0 or 1—but we often think probabilities for such events have nonextreme values.

This brings us to another problem for frequency theories. Suppose I have a penny, and think that if I flip it, the probability that the flip will come out heads is 1/2. Let's just grant *arguendo* that the correct reference class for this event is penny flips. According to the frequency theory, the probability that this flip will come up heads is the fraction of all penny flips that ever occur which come up heads. Yet while I'd be willing to bet that fraction is *close* to 1/2, I'd be willing to bet even more that the fraction is not *exactly* 1/2. (For one thing, the number of penny flips that will ever occur in the history of the universe might be an odd number!) For any finite run of trials of a particular event-type, it seems perfectly coherent to imagine—indeed, to *expect*—that a particular outcome will occur with a frequency not precisely equal to that outcome's probability. Yet if the frequency theory is correct, this is *conceptually impossible* when the run in question encompasses every event trial that will ever occur.

One might respond that the probability of heads on the flip of a penny is not the frequency with which penny flips *actually* come up heads over the finite history of our universe; instead, it's the frequency *in the limit*—were pennies to continue being flipped forever. This gives us **hypothetical frequency theory**, on which the probability of an outcome is the frequency it would approach were the event repeated indefinitely. Yet this move undermines one of the original appeals of the frequency approach: its empiricism. The proportion of event repetitions that produce a particular outcome in the actual world is the sort of thing that could be *observed* (at least in principle)—providing a sound empirical base for otherwise-mysterious "probability" talk. Empirically grounding hypothetical frequencies is a much more difficult task.

Moreover, there seem to be events that *couldn't* possibly be repeated many many times, and even events that couldn't be repeated once. Before the Large Hadron Collider was switched on, physicists were asked for the probability that doing so would destroy the Earth. Were that to have happened, switching on the Large Hadron Collider would not have been a repeatable event. Scientists also sometimes discuss the probability that our universe began with a Big Bang; arguably, that's not an event that will happen over and over or even *could* happen over and over. So it's difficult to understand talk about how frequently the universe would begin with a Bang were the number of times the universe started increased toward the limit. This problem of assigning meaningful nonextreme probabilities to individual, perhaps non-repeatable events is called the **problem of the single case**.

The frequentist still has moves available. Faced with a single event that's non-repeatable in the actual world, she might ask what proportion of times that event produces a particular outcome across *other* possible worlds.[3] But now the prospects for analyzing "probability" talk in empirically observable terms have grown fairly dim.

An alternate interpretation of probability admits that probabilities are related to frequencies, but draws our attention to the features that *cause* particular outcomes to appear with the frequencies that they do. What is it about a penny that makes it come up heads about half the time? Presumably something about its physical attributes, the symmetries with which it interacts with surrounding air as it flips, etc. These traits lend the penny a certain tendency to come up heads, and an equal tendency to come up tails. This quantifiable disposition—or **propensity**—would generate certain frequencies were a long run of trials to be staged. But the propensity is also at work in each individual flip, whether that flip is ever repeated or could ever be repeated.

A non-repeatable experimental setup may possess a nonextreme propensity to generate a particular outcome.

While an early propensity theory appeared in the work of Charles Sanders Peirce (1910/1932), propensity's most famous champion was Karl Popper (1957). Popper was especially motivated by developments in quantum mechanics. In quantum theory the Born rule calculates probabilities of experimental outcomes from a particular quantity (the amplitude of the wave-function) with independent significance in the theory's dynamics. Moreover, this quantity can be determined for a particular experimental setup even if that setup is never to be repeated (or *couldn't* be repeated) again. This gives propensities a respectable place within an empirically established scientific theory. Propensities may also figure in such theories as statistical mechanics and population genetics.

Yet even if there are propensities in the world, it seems difficult to interpret *all* probabilities as propensities. Suppose we're discussing the likelihood that a particular outcome occurs given that a quantum experiment is set up in a particular fashion. This is a conditional probability, and it has a natural interpretation in terms of physical propensities: the experimental setup described in the condition of the conditional probability has a particular causal tendency to produce the outcome. But where there is a likelihood, probability mathematics suggests there will also be a posterior—if there's a probability of outcome given setup, there should also be a probability of setup given outcome. Yet the latter hardly makes sense as a physical propensity—does an experimental outcome have a quantifiable causal tendency to produce the particular experimental setup from which it results?[4]

Some philosophers—especially those of a Humean bent—are also suspicious of the metaphysics of propensities. From their point of view, causes are objectionable enough; even worse to admit propensities that seem to be a kind of graded causation. Nowadays most philosophers of science agree that we need some notion of physical probability that applies to the single case. Call this notion **objective chance**. But whether objective chances are best understood via propensity theory, a "best systems" analysis (Lewis 1994), or some other approach is a hotly contested matter.

Finally, whatever objective chances turn out to be, they are governed by the physical laws of our world. That means there can be no objective chance that the physical laws are one way or another. (What set of laws beyond the physical might determine such chances?) Yet it seems physicists can meaningfully discuss the probability that the physical laws of the universe will turn out to be such-and-such. While the notion of objective chance makes sense of *some* of our "probability" talk, it nevertheless seems to leave a remainder.

5.1.2 Two distinctions in Bayesianism

So what *are* physicists talking about when they discuss the probability that the physical laws of the universe are one way rather than another? Perhaps they are expressing their degrees of confidence in alternative physical hypotheses. Perhaps there are no probabilities out in the world, independent of us, about which our opinions change as we gain evidence. Instead, it may be that facts in the world are simply true or false, probability-free, and "probability" talk records our changing confidences in those facts in the face of changing evidence.

Bayesian theories are often characterized as "Subjective" or "Objective", but this terminology can be used to draw two different distinctions. One of them concerns the interpretation of "probability" talk. On this distinction—which I'll call the **semantic distinction**—Subjective Bayesians adopt the position I proposed in the previous paragraph. For them, "probability" talk expresses or reports the degrees of confidence of the individuals doing the talking, or perhaps of communities to which they belong. Objective Bayesians, on the other hand, interpret "probability" assertions as having truth-conditions independent of the attitudes of particular agents or groups of agents.[5] In the twentieth century, talk of "Objective" and "Subjective" Bayesianism was usually used to draw this semantic distinction.[6]

More recently the "Subjective Bayesian/Objective Bayesian" terminology has been used to draw a different distinction, which I will call the **normative distinction**. However we interpret the meaning of "probability" talk, we can grant that agents assign different degrees of confidence to different propositions (or, more weakly, that it is at least useful to model agents as if they do). Once we grant that credences exist and are subject to rational constraints, we may inquire about the strength of those constraints.

On one end of the spectrum, Objective Bayesians (in the normative sense) endorse what Richard Feldman (2007) and Roger White (2005) have called the

Uniqueness Thesis: Given any proposition and body of total evidence, there is exactly one attitude it is rationally permissible for agents with that body of total evidence to adopt toward that proposition.

Assuming the attitudes in question are degrees of belief, the Uniqueness Thesis says that given any evidential situation, there's exactly one credence that any agent is rationally permitted to adopt toward a given proposition in that situation. The Uniqueness Thesis entails **evidentialism**, according to which the attitudes rationally permissible for an agent supervene on her evidence.

Suppose we have two agents with identical total evidence who adopt different credences toward some propositions. Because Objective Bayesians (in the normative sense) endorse the Uniqueness Thesis, they will maintain that at least one of these agents is responding to her evidence irrationally. In most real-life situations, different agents have different bodies of total evidence—and even different bodies of *relevant* evidence—so many discrepancies in their attitudes can be chalked up to evidential differences. But we have stipulated in this case that the agents have identical evidence, so whatever causes the differences in their attitudes, it can't be the contents of their evidence. In Section 4.3 we identified the extra-evidential factors that determine an agent's attitudes in light of her total evidence as her "ultimate epistemic standards". These epistemic standards might reflect pragmatic influences, a predilection for hypotheses with certain features, a tendency toward mistrust or skepticism, etc.

The Hypothetical Priors Theorem tells us that whenever an agent's credence distributions over time satisfy the probability axioms, Ratio Formula, and Conditionalization, her epistemic standards can be represented by a hypothetical prior distribution. This regular, probabilistic distribution stays constant as the agent gains evidence over time. Yet we can always recover the agent's credence distribution at a given time by conditionalizing her hypothetical prior on her total evidence at that time.

The core Bayesian rules (probability axioms, Ratio Formula, Conditionalization) leave a wide variety of hypothetical priors available. Assuming they satisfy the core rules, our two agents who assign different credences in response to the same total evidence must have different hypothetical priors. According to the Objective Bayesian (in the normative sense), any time such a situation arises at least one of the agents must be violating rational requirements. Thus the Objective Bayesian thinks there is exactly one set of rationally permissible hypothetical priors—one set of correct epistemic standards embodying rational agents' common responses to evidence.

How might the unique rational hypothetical prior be generated, and how might we justify the claim that it is uniquely correct? Our *ongoing* epistemic standards for responding to new pieces of empirical evidence are often informed by other pieces of evidence we gained in the past. I react to a fire alarm in a particular way because I've experienced such alarms before; one piece of evidence helps determine how we interpret the next. But *ultimate* epistemic standards—the ones represented by our hypothetical priors—dictate responses to our *total* evidence, and so must be rationally antecedent to all of our evidence. If we are to select and justify a unique set of ultimate epistemic standards, we must do so a priori.

Extending a tradition that dated back to Bolzano (1837/1973) and perhaps even Leibniz,[7] Keynes (1921) and Carnap (1950) argued that just as there are objective facts about which propositions are logically *entailed* by a given body of evidence, there are objective logical facts about the degree to which a body of evidence probabilifies a particular proposition. Carnap went on to offer a mathematical algorithm for calculating the unique logical hypothetical priors from which these facts could be determined; we will discuss that algorithm in Chapter 6. (The **logical interpretation** of probability holds that an agent's "probability" talk concerns logical probabilities relative to her current total evidence.)[8] Many recent theorists, while backing away from Keynes's and Carnap's position that these values are *logical*, nevertheless embrace the idea of **evidential probabilities** reflecting the degree to which a proposition is probabilified by a given body of evidence. If you think that rationality requires an agent to assign credences equal to the unique, true evidential probabilities on her current total evidence, you have an Objective Bayesian view in the normative sense.[9]

At the other end of the spectrum from Objective Bayesians (in the normative sense) are theorists who hold that the probability axioms and Ratio Formula are the only rational constraints on hypothetical priors.[10] The literature often defines "Subjective Bayesians" as people who hold this view. But that terminology leaves no way to describe theorists in the middle of the spectrum—the vast majority of Bayesian epistemologists who believe in rational constraints on hypothetical priors that go beyond the core rules but are insufficient to narrow things down to a single permissible standard. I will use the term "Subjective Bayesian" (in the normative sense) to refer to anyone who thinks more than one hypothetical prior is rationally permissible. I will call people who think the Ratio Formula and probability axioms are the only rational constraints on hypothetical priors "extreme Subjective Bayesians".

Subjective Bayesians allow for what White calls **permissive cases**: situations in which two agents reach different conclusions on the basis of the same total evidence without either party's making a rational mistake. This is because each agent interprets the evidence according to different (yet rationally acceptable) epistemic standards, which allow them to draw different conclusions.

I have distinguished the semantic and normative Objective/Subjective Bayesian distinctions because they can cross-cut one another. Historically, Ramsey (1931) and de Finetti (1931/1989) reacted to Keynes's Objective Bayesianism with groundbreaking theories that were Subjective in both the semantic and normative senses. But one could be a Subjective Bayesian in the semantic sense—taking agents' "probability" talk to express their own current credences—while maintaining that strictly speaking only one credence distribution is rationally permitted in each situation (thereby adhering to Objective Bayesianism in the normative sense). Going in the other direction, one could admit the existence of degrees of belief while holding that they're not what "probability" talk concerns. This would give an Objective Bayesian semantic view that combined with either Subjective or Objective Bayesianism in the normative sense. Finally, probability semantics need not be monolithic; many Bayesians now hold that some "probability" assertions in everyday life express credences, others report objective chances, and still others indicate what would be reasonable to believe given one's evidence.[11]

Regardless of her position on the semantics, any Bayesian who isn't an extreme Subjective Bayesian in the normative sense will concede that there are rational constraints on agents' hypothetical priors beyond the probability axioms and Ratio Formula. The rest of this chapter investigates what some of those additional constraints might be. I should note at the outset, though, that the more powerful and widely applicable these constraints get, the more they seem to be beset by problems. Many Subjective Bayesians (in the normative sense) would be happy to adopt an Objective position, if only they could see past the numerous shortcomings of the principles Objective Bayesians propose to generate unique hypothetical priors. Richard Jeffrey characterized his Subjective Bayesian position as follows:

> As a practical matter, I think one can give necessary conditions for reasonableness of a set of partial beliefs that go beyond mere [probabilistic] coherence—in special cases. The result is a patch-work quilt, where the patches have frayed edges, and there are large gaps where we lack patches altogether. It is not the sort of seamless garment philosophers like to wear; but (we ragged pragmatists say), the philosophers are naked! Indeed we have no proof that no more elegant garb than our rags is available, or ever will be, but we haven't seen any, yet, as far as we know. We will be the first to snatch it off the racks, when the shipments come in. But perhaps they never will. Anyway, for the time being, we are dressed in rags, tied neatly at the waist with a beautiful cord—probabilistic coherence. (It is the only cord that visibly distinguishes us from the benighted masses.) (1970, p. 169)

5.2 Deference principles

5.2.1 The Principal Principle

Bayesian epistemology concerns agents' degrees of belief. Yet most contemporary Bayesian epistemologists also believe that the world contains objective chances of some sort—physical probabilities that particular events will produce particular outcomes. This raises the question of how subjective credences and objective chances should relate. One obvious response is a principle of direct inference: roughly, rational agents set their credences in line with what they know of the chances. If you're certain a die is fair (has an equal objective chance of landing on each of its faces), you should assign equal credence to each possible roll outcome.

While direct inference principles have a long history, the most famous such principle relating credence and chance is David Lewis's (1980) Principal Principle. The Principal Principle's most straightforward consequence is that if you are certain an event has objective chance x of producing a particular outcome, and you have no other information about that event, then your credence that the outcome will occur should be x. For many Bayesian purposes this is all one needs to know about the Principal Principle. But in fact the Principle is a more finely honed instrument, because Lewis wanted it to deal with complications like the following: (1) What if you're uncertain about the objective chance of the outcome? (2) What if the outcome's chance changes over time? (3) What if you have additional information about the event besides what you know of the chances? The rest of this section explains how the Principal Principle deals with those eventualities. If you're not interested in the details, feel free to skip to Section 5.2.2.

So: Suppose it is now 1 p.m. on a Monday. I tell you that over the weekend I found a coin from a foreign country that is somewhat irregular in shape. Despite being foreign, one side of the coin is clearly the "Heads" side and the other is "Tails". I also tell you that I flipped the foreign coin today at noon.

Let H be the proposition that the noon coin flip landed heads. Consider each of the propositions below one at a time, and decide what your credence in H would be if that proposition was *all* you knew about the coin in addition to the information in the previous paragraph:

E_1: After discovering the coin I spent a good part of my weekend flipping it, and out of my 100 weekend flips sixty-four came up heads.

E_2: The coin was produced in a factory that advertises its coins as fair, but also has a side business generating black-market coins biased toward tails.

E_3: The coin is fair (has a 1/2 chance of landing heads).

E_4: Your friend Amir was with me at noon when I flipped the coin, and he told you it came up heads.

Hopefully it's fairly clear how to respond to each of these pieces of evidence, taken singly. For instance, in light of the frequency information in E_1, it seems rational to have a credence in H somewhere around 0.64. We might debate whether precisely 0.64 is required,[12] but certainly a credence in H of 0.01 (assuming E_1 is your *only* evidence about the coin) seems unreasonable.

This point generalizes to a rational principle that whenever one's evidence includes the frequency with which events of type A have produced outcomes of type B, one should set one's credence that the next A-event will produce a B-outcome equal to (or at least in the vicinity of) that frequency.[13] While some version of this principle ought to be right, working out the specifics creates problems like those faced by the frequency interpretation of probability. For instance, we have a reference class problem: Suppose my evidence includes accident frequency data for drivers in general, for sixteen-year-old drivers in general, and for my sixteen-year-old daughter in particular. Which value should I use to set my credence that my daughter will get in a car accident tonight? The more specific data seems more relevant, but the more general data reflects a larger sample.[14]

There are statistical tools available for dealing with these problems, some of which we will discuss in Chapter 13. But for now let's focus on a different question about frequency data: *Why* do we use known flip outcomes to predict the outcome of unobserved flips? Perhaps because known outcomes indicate something about the physical properties of the coin itself; they help us figure out its objective chance of coming up heads. Known flip data influence our unknown flip predictions because they make us think our coin has a particular chance profile. In this case, frequency data influences predictions *by way of* our opinions about objective chances.

This relationship between frequency and chance is revealed when we *combine* pieces of evidence listed above. We've already said that if your only evidence about the coin is E_1—it came up heads on sixty-four of 100 known tosses—then your credence that the noon toss (of uncertain outcome) came up heads should be around 0.64. On the other hand, if your only evidence is E_3, that the coin is fair, then I hope it's plausible that your credence in H should be 0.5. But what if you're already certain of E_3, and then learn E_1? In that case your credence in heads should still be 0.5.

Keep in mind we're imagining you're *certain* that the coin is fair before you learn the frequency data; we're not concerning ourselves with the possibility that, say, learning about the frequencies makes you suspicious of the source from which you learned that the coin is fair. If it's a fixed, unquestionable truth for you that the coin is fair, then learning that it came up sixty-four heads on 100 flips will not change your credence in heads. If *all* you had was the frequency information, that would support a different hypothesis about the chances. But it's not as if sixty-four heads on 100 flips is *inconsistent* with the coin's being fair—a fair coin usually won't come up heads on exactly half the flips in a given sample. So once you're already certain of heads, the frequency information becomes redundant, irrelevant to your opinions about unknown flips. Frequencies help you learn about chances, so if you are already certain of the chances there's nothing more for frequency information to do.

David Lewis called information that can change your credences about an event only *by way of* changing your opinions about its chances **admissible** information. His main insight about admissible information was that when the chance values for an event have already been established, admissible information becomes irrelevant to a rational agent's opinions about the outcome.

Here's another example: Suppose your only evidence about the noon flip outcome is E_2, that the coin was produced in a factory that advertises its coins as fair but has a side business in tails-biased coins. Given only this information your credence in H should be somewhere below 0.5. (Exactly how far below depends on how extensive you estimate the side business to be.) On the other hand, suppose you learn E_2 after already learning E_3, that this particular coin is fair. In that context, E_2 becomes irrelevant, at least with respect to predicting flips of this coin. E_2 is relevant in isolation because it informs you about the chances associated with the coin. But once you're certain that the coin is fair, information E_2 only teaches you that you happened to get lucky not to have a black-market coin; it doesn't do anything to push your credence in H away from 0.5. E_2 is admissible information.

Contrast that with E_4, your friend Amir's report that he observed the flip landing heads. Assuming you trust Amir, E_4 should make you highly confident in H. And this should be true even if you already possess information E_3 that the coin is fair. Notice that E_3 and E_4 are consistent; the coin's being fair is consistent with its having landed heads on this particular flip, and with Amir's reporting that outcome. But E_4 trumps the chance information; it moves your credence in heads away from where it would be (0.5) if you knew only E_3. Information about this particular flip's outcome does not change your credences about the flip *by way of* influencing your opinions about the chances. You still think the coin is fair, and was fair at the time it was flipped. You just know now that the fair coin happened to come up heads on this occasion. Information about this flip's outcome is inadmissible with respect to H.

Lewis expressed his insight about the irrelevance of admissible information in his famous chance-credence principle, the

Principal Principle: Let Pr_H be any reasonable initial credence function. Let t_i be any time. Let x be any real number in the unit interval. Let $\mathrm{Ch}_i(A) = x$ be the proposition that the chance, at time t_i, of A's holding equals x. Let E be any proposition compatible with $\mathrm{Ch}_i(A) = x$ that is admissible at time t_i. Then

$$\mathrm{Pr}_H(A \mid \mathrm{Ch}_i(A) = x \,\&\, E) = x$$

(I have copied this principle verbatim from Lewis 1980, p. 266, though I have altered Lewis's notation to match our own.) There's a lot to unpack in the Principal Principle, so we'll take it one step at a time. First, Lewis's "reasonable initial credence function" sounds a lot like an initial prior distribution. Yet we saw in Section 4.3 that the notion of an initial prior is problematic, and there are passages in Lewis that make it sound more like he's talking about a hypothetical prior.[15] So I will interpret the "reasonable initial credence function" as your hypothetical prior distribution, and designate it with our notation "Pr_H".

The Principal Principle is proposed as a rational constraint on hypothetical priors, one that goes beyond the probability axioms and Ratio Formula. Why frame the Principal Principle around hypothetical priors, instead of focusing on the credences of rational agents at particular times? One advantage of the hypothetical-priors approach is that it makes the total evidence at work explicit, and therefore easy to reference in the principle. Recall from Section 4.3 that a hypothetical prior is a probabilistic, regular distribution containing no contingent evidence. A rational agent is associated with a particular hypothetical prior, in the sense that if you conditionalize that hypothetical prior on the agent's total evidence at any given time, you get the agent's credence distribution at that time.

In the Principal Principle we imagine that a real-life agent is considering some proposition A about the outcome of a chance event. She has

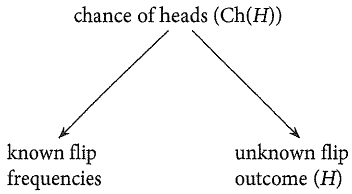

Figure 5.1 Chances screen off frequencies

some information about the chance of A, $\text{Ch}_i(A) = x$, and then some further evidence E. So her total evidence is $\text{Ch}_i(A) = x \,\&\, E$, and by the definition of a hypothetical prior her credence in A equals $\text{Pr}_H(A \mid \text{Ch}_i(A) = x \,\&\, E)$. Lewis claims that as long as E is both admissible for A, and is compatible (which we can take to mean "logically consistent") with $\text{Ch}_i(A) = x$, E should make no difference to the agent's credence in A. In other words, as long as E is admissible and compatible, the agent should be just as confident in A as she would be if all she knew were $\text{Ch}_i(A) = x$. That is, her credence in A should be x.

Return to our example about the noon coin flip, and the relationship between chance and frequency information. Suppose that at 1 p.m. your total evidence about the flip outcome consists of E_1 and E_3. E_3, the chance information, says that $\text{Ch}(H) = 0.5$. E_1, the frequency information, comprises the rest of your total evidence, which will play the role of E in the Principal Principle. Because this additional evidence is both consistent with $\text{Ch}(H) = 0.5$ and admissible for H, the Principal Principle says your 1 p.m. credence in H should be 0.5. Which is exactly the result we came to before.

We can gain further insight into this result by connecting it to our earlier (Section 3.2.4) discussion of causation and screening off. Figure 5.1 illustrates the causal relationships in the coin example between chances, frequencies, and unknown results. The coin's physical structure, associated with its objective chances, causally influenced the frequency with which it came up heads in the previous trials. The coin's physical makeup also affects the outcome of the unknown flip. Thus previous frequency information is relevant to the unknown flip, but only by way of the chances.[16] We saw in Section 3.2.4 that when this kind of causal fork structure obtains, the common cause screens its effects off from each other.[17] Conditional on the chances, frequency information becomes irrelevant to flip predictions. That is,

$$\text{Pr}_H(H \mid \text{Ch}(H) = 0.5 \,\&\, E) = \text{Pr}_H(H \mid \text{Ch}(H) = 0.5) \qquad (5.1)$$

and intuitively the expression on the right should equal 0.5.

A similar analysis applies if your total evidence about the coin flip contains only $\text{Ch}(H) = 0.5$ and E_2, the evidence about the coin factory. This time

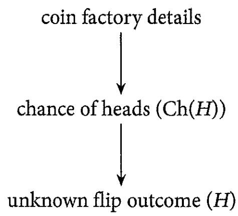

Figure 5.2 Chance in a causal chain

our structure is a causal chain, as depicted in Figure 5.2. The situation in the coin factory causally affects the chance profile of the coin, which in turn causally affects the unknown flip outcome. Thus the coin factory information affects opinions about H by way of the chances, and if the chances are already determined then factory information becomes irrelevant. Letting the factory information play the role of E in the Principal Principle, the chances screen off E from H and we have the relation in Equation (5.1).

Finally, information E_4, your friend Amir's report, is not admissible information about H. E_4 affects your opinions about H, but not by way of affecting your opinions about the chances. The Principal Principle applies only when E, the information possessed in addition to the chances, is admissible. Since E_4 is inadmissible, the Principal Principle supplies no guidance about setting your credences in light of it.

There are still a few details in the principle to unpack. For instance, the chance expression $\mathrm{Ch}_i(A)$ is indexed to a time t_i. That's because the chance that a particular proposition will obtain can change as time goes on. For instance, suppose that at 11am our foreign coin was fair, but at 11:30 I stuck a particularly large, non-aerodynamic wad of chewing gum to one of its sides. In that case, the proposition H that the coin comes up heads at noon would have a chance of 0.5 at 11am but might have a different chance after 11:30. The physical details of an experimental setup determine its chances, so as physical conditions change chances may change as well.[18]

Finally, the Principal Principle's formulation in terms of conditional credences allows us to apply it even when an agent doesn't have full information about the chances. Suppose your total evidence about the outcome A of some chance event is E. E influences your credences in A by way of informing you about A's chances (so E is admissible), but E does not tell you what the chances are exactly. Instead, E tells you that the chance of A (at some specific time, which I'll suppress for the duration of this example) is either 0.7 or 0.4. E also supplies you with a favorite among these two chance hypotheses: it sets your credence that 0.7 is the true chance at 2/3, and your credence that 0.4 is the true chance at 1/3.

How can we analyze this situation using the Principal Principle? Since your total evidence is E, the definition of a hypothetical prior distribution tells us that your current credences cr should be related to your hypothetical prior Pr_H as follows:

$$\mathrm{cr}(A) = \mathrm{Pr}_H(A \mid E) \tag{5.2}$$

This value is not dictated directly by the Principal Principle. However, the Principal Principle does set

$$\mathrm{Pr}_H(A \mid \mathrm{Ch}(A) = 0.7 \;\&\; E) = 0.7 \tag{5.3}$$

because we stipulated that E is admissible. Similarly, the Principal Principle sets

$$\mathrm{Pr}_H(A \mid \mathrm{Ch}(A) = 0.4 \;\&\; E) = 0.4 \tag{5.4}$$

Since E narrows the possibilities down to two mutually exclusive chance hypotheses, those hypotheses ($\mathrm{Ch}(A) = 0.7$ and $\mathrm{Ch}(A) = 0.4$) form a partition relative to E. Thus we can apply the Law of Total Probability (in its conditional credence form)[19] to obtain

$$\begin{aligned} \mathrm{Pr}_H(A \mid E) = {} & \mathrm{Pr}_H(A \mid \mathrm{Ch}(A) = 0.7 \;\&\; E) \cdot \mathrm{Pr}_H(\mathrm{Ch}(A) = 0.7 \mid E) + \\ & \mathrm{Pr}_H(A \mid \mathrm{Ch}(A) = 0.4 \;\&\; E) \cdot \mathrm{Pr}_H(\mathrm{Ch}(A) = 0.4 \mid E) \end{aligned} \tag{5.5}$$

By Equations (5.3) and (5.4), this is

$$\mathrm{Pr}_H(A \mid E) = 0.7 \cdot \mathrm{Pr}_H(\mathrm{Ch}(A) = 0.7 \mid E) + 0.4 \cdot \mathrm{Pr}_H(\mathrm{Ch}(A) = 0.4 \mid E) \tag{5.6}$$

As Equation (5.2) suggested, $\mathrm{Pr}_H(\cdot \mid E)$ is just $\mathrm{cr}(\cdot)$. So this last equation becomes

$$\mathrm{cr}(A) = 0.7 \cdot \mathrm{cr}(\mathrm{Ch}(A) = 0.7) + 0.4 \cdot \mathrm{cr}(\mathrm{Ch}(A) = 0.4) \tag{5.7}$$

Finally, we fill in the values stipulated in the problem to conclude

$$\mathrm{cr}(A) = 0.7 \cdot 2/3 + 0.4 \cdot 1/3 = 0.6 \tag{5.8}$$

That's a lot of calculation, but the overall lesson comes to this: When your total evidence is admissible and restricts you to a finite set of chance values for A, the Principal Principle sets your credence in A equal to a weighted average of those chance values (where each chance value is weighted by your credence that it's the true chance).

This is an extremely useful conclusion, *provided* we can discern what kinds of evidence are admissible. Lewis writes that, "Admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about the chances of those outcomes" (1980, p. 272). He then sketches out some categories of information that we should expect to be admissible, and inadmissible. For example, evidence about events causally upstream from the chances will be admissible; such events will form the first link in a causal chain like Figure 5.2. This includes information about the physical laws that give rise to chances—information that affects our credences about experimental outcomes by affecting our views about their chances. On the other hand, evidence about effects of the chance outcome is inadmissible, as we saw in the example of Amir's report. Generally, then, it's a good rule of thumb that facts concerning events temporally *before* the chance outcome are admissible, and inadmissible information is always about events *after* the outcome. (Though Lewis does remark at one point (1980, p. 274) that if backward causation is possible, seers of the future or time-travelers might give us inadmissible information about chance events yet to come.)

We'll close our discussion of the Principal Principle with a couple of caveats.[20] First, I have been talking about coin flips, die rolls, etc. as if their outcomes have non-extreme objective chances. If you think that these outcomes are fully determined by the physical state of the world prior to such events, you might think these examples aren't really chancy at all—or if there are chances associated with their outcomes, the world's determinism makes those chances either 1 or 0. There are authors who think non-extreme chance assignments are compatible with an event's being fully deterministic. This will be especially plausible if you think a single phenomenon may admit of causal explanations at multiple levels of description. (Though the behavior of a gas sample is fully determined by the positions and velocities of its constituent particles, we might still apply a statistical thermodynamics that treats the sample's behavior as chancy.) In any case, if the compatibility of determinism and non-extreme chance concerns you, you can replace my coin-flipping and die-rolling examples with genuinely indeterministic quantum events.

Second, you might think frequency data can affect rational credences without operating through opinions about chances. Suppose a new patient walks into a doctor's office, and the doctor assigns a credence that the patient has a particular disease equal to that disease's frequency in the general population.

In order for this to make sense, must the doctor assume that physical chances govern who gets the disease, or that the patient was somehow brought to her through a physically chancy process? (That is, must the frequency affect the doctor's credences by informing her opinions about chances?) This will depend on how broadly we are willing to interpret macroscopic events as having objective chances. But unless chances are literally everywhere, inferences governed by the Principal Principle form a proper subset of the legitimate instances of inductive reasoning. To move from frequencies in an observed population to predictions about the unobserved when chances are not present, we may need something like the frequency-credence principle (perhaps made more plausible by incorporating statistical tools) with which this section began. Or we may need a theory of inductive confirmation in general—something we will try to construct in Chapter 6.

For the time being, the message of the Principal Principle is clear: Where there are objective chances in the world, we should align our credences with them to the extent we can determine what they are. While there are exceptions to this rule, they can be worked out by thinking about the causal relations between our information and the chances of which we're aware.

5.2.2 Expert principles and Reflection

The Principal Principle is sometimes described as a **deference principle**: to the extent you can determine what the objective chances are, the principle directs you to defer to them by making your credences match. In a certain sense, you treat the chances as authorities on what your credences should be. Might other sorts of authorities demand such rational deference?

Testimonial evidence plays a large role in how we learn about the world. Suppose an expert on some subject reveals her credences to you. Instead of coming on television and talking about the "probability" of snow, the weather forecaster simply tells you she's 30% confident that it will snow tomorrow. It seems intuitive that—absent other evidence about tomorrow's weather—you should set your credence in snow to 0.30 as well.

We can generalize this intuition with a principle for deference to experts modeled on the Principal Principle:

$$\mathrm{Pr}_H(A \mid \mathrm{cr}_E(A) = x) = x \tag{5.9}$$

Here $\Pr_H$ is a rational agent's hypothetical prior distribution, representing her ultimate epistemic standards for assigning attitudes on the basis of total evidence. A is a proposition within some particular subject matter, and $\text{cr}_E(A) = x$ is the proposition that an expert on that subject matter assigns credence x to A. As we've discussed before (Section 4.3), an agent's credences at a given time equal her hypothetical prior conditionalized on her total evidence at that time. So Equation (5.9) has consequences similar to the Principal Principle's: When a rational agent is *certain* that an expert assigns credence x to A, and that fact constitutes her total evidence relevant to A, satisfying Equation (5.9) will leave her with an unconditional credence of $\text{cr}(A) = x$. On the other hand, an agent who is uncertain of the expert's opinion can use Equation (5.9) to calculate a weighted average of all the values she thinks the expert might assign.[21]

Equation (5.9) helps us figure out how to defer to someone we've identified as an expert. But it doesn't say anything about how to make that identification! Ned Hall helpfully distinguishes two kinds of experts we might look for:

> Let us call the first kind of expert a *database-expert*: she earns her epistemic status simply because she possesses more information. Let us call the second kind an *analyst-expert*: she earns her epistemic status because she is particularly good at evaluating the relevance of one proposition to another.
>
> (2004, p. 100)

A **database expert**'s evidence (or at least, her evidence relevant to the matter at hand) is a superset of mine. While she may not reveal the contents of her evidence, I can still take advantage of it by assigning the credences she assigns on its basis. On the other hand, I defer to an **analyst expert** not because she has superior evidence but because she is particularly skilled at forming opinions from the evidence we share. Clearly these categories can overlap; relative to me, a weather forecaster is probably both an analyst expert and a database expert with respect to the weather.

One particular database expert has garnered a great deal of attention in the deference literature: an agent's future self. Because Conditionalization retains certainties (Section 4.1.1), at any given time a conditionalizing agent will possess all the evidence possessed by each of her past selves—and typically quite a bit more. So an agent who is certain she will update by conditionalizing should treat her future self as a database expert.[22] On the supposition that her future self will assign credence x to a proposition A, she should now assign credence x to A as well. This is van Fraassen's (1984)

Reflection Principle: For any proposition A in $\mathcal{L}$, real number x, and times t_i and t_j with $j > i$, rationality requires

$$\text{cr}_i(A \mid \text{cr}_j(A) = x) = x$$

Although the Reflection Principle mentions both the agent's t_i and t_j credences, strictly speaking it is a *synchronic* principle, relating various credences the agent assigns at t_i. If we apply the Ratio Formula and then cross-mutiply, Reflection gives us:

$$\text{cr}_i[A \ \& \ \text{cr}_j(A) = x] = x \cdot \text{cr}_i[\text{cr}_j(A) = x] \tag{5.10}$$

The two credences related by this equation are both assigned *at* t_i; they just happen to be credences *in* some propositions about t_j.

Despite this synchronic nature, Reflection bears an intimate connection to Conditionalization. If an agent is certain she will update by conditionalizing between t_i and t_j—and meets a few other side conditions—Reflection follows. For instance, the Reflection Principle can be proven from the following set of conditions:

1. The agent is certain at t_i that cr_j will result from conditionalizing cr_i on the total evidence she learns between t_i and t_j (call it E).
2. The agent is certain at t_i that E (whatever it may be) is true.
3. $\text{cr}_i(\text{cr}_j(A) = x) > 0$
4. At t_i the agent can identify a set of propositions S in $\mathcal{L}$ such that:
   (a) The members of S form a partition relative to the agent's certainties at t_i.
   (b) At t_i the agent is certain that E is one of the propositions in S.
   (c) For each member of S, the agent is certain at t_i what cr_i-value she assigns to A conditional on that member.

References to a proof can be found in the Further Reading. Here I'll simply provide an example that illustrates the connection between Conditionalization and Reflection. Suppose that I've rolled a die you're certain is fair, but as of t_1 have told you nothing about the outcome. However, at t_1 you're certain that between t_1 and t_2 I'll reveal to you whether the die came up odd or even. The Reflection Principle suggests you should assign

$$\text{cr}_1(6 \mid \text{cr}_2(6) = 1/3) = 1/3 \tag{5.11}$$

Assuming the enumerated conditions hold in this example, we can reason to Equation (5.11) as follows: In this case the partition S contains the proposition that the die came up odd and the proposition that it came up even. You are certain at t_1 that one of these propositions will provide the E you learn before t_2. You're also certain that your $\text{cr}_2(6)$ value will result from conditionalizing your t_1 credences on E. So you're certain at t_1 that

$$\text{cr}_2(6) = \text{cr}_1(6 \mid E) \tag{5.12}$$

Equation (5.11) involves your t_1 credence in 6 conditional on the supposition that $\text{cr}_2(6) = 1/3$. To determine this value, let's see what conditional reasoning you could do at t_1, not yet certain what credences you will actually assign at t_2, but temporarily supposing that $\text{cr}_2(6) = 1/3$. We just said that at t_1 you're certain of Equation (5.12), so given the supposition you can conclude that $\text{cr}_1(6 \mid E) = 1/3$. Then you can examine your current t_1 credences conditional on both odd and even, and find that $\text{cr}_1(6 \mid E)$ will equal 1/3 only if E is the proposition that the die came up even. (Conditional on the die's coming up odd, your credence in a 6 would be 0.) Thus you can conclude that E is the proposition that the die came up even. You're also certain at t_1 that E (whatever its content) is true, so concluding that E says the die came up even allows you to conclude that the die did indeed come up even. And on the condition that the die came up even, your t_1 credence in a six is 1/3.

All of the reasoning in the previous paragraph was conditional, starting with the supposition that $\text{cr}_2(6) = 1/3$. We found that conditional on this supposition, your rational credence in six would be 1/3. And that's exactly what the Reflection Principle gave us in Equation (5.11).[23] Information about your future credences tells you something about what evidence you'll receive between now and then. And information about what evidence you'll receive in the future should be incorporated into your credences in the present.

But how often do we really get information about our future opinions? Approached the way I've just done, the Reflection Principle seems to have little real-world applicability. But van Fraassen originally proposed Reflection in a very different spirit. He saw the principle as stemming from basic commitments we undertake when we form opinions.

van Fraassen drew an analogy to making promises. Suppose I make a promise at a particular time, but at the same time admit to being unsure whether I will actually carry it out. van Fraassen writes that "To do so would mean that I am now less than fully committed (a) to giving due regard to the felicity conditions for this act, or (b) to standing by the commitments I shall overtly enter" (1984, p. 255). To fully stand behind a promise requires full confidence that you will carry it out. And what goes for current promises goes for future promises as well: if you know you'll make a promise later on, failing to be fully confident *now* that you'll enact the future promise is a betrayal of solidarity with your future promising self.

Now apply this lesson to the act of making judgments: assigning a different credence *now* to a proposition than the credence you know you'll assign in the future is a failure to stand by the commitments implicit in that future opinion. As van Fraassen puts it in a later publication, "Integrity requires me to express my commitment to proceed in what I now classify as a rational manner, to stand behind the ways in which I shall revise my values and opinions" (1995, pp. 25–6). This is his motivation for endorsing the Reflection Principle.[24] For van Fraassen, Reflection brings out a substantive commitment inherent in judgment, which underlies various other rational requirements. For instance, since van Fraassen's argument for Reflection does not *rely* on Conditionalization, van Fraassen at one point (1999) uses Reflection to *argue* for Conditionalization!

Of course, one might not agree with van Fraassen that assigning a credence involves such strong commitments. And even if Reflection can be supported as van Fraassen suggests, moving from that principle to Conditionalization is going to require substantive further premises. As we've seen, Reflection is a synchronic principle, relating an agent's attitudes at one time to other attitudes she assigns at the same time. By itself, Reflection may support a conclusion to the effect that an agent with certain attitudes at a given time is required to *predict* that she will update by Conditionalization. But to actually establish Conditionalization as a diachronic norm, we would need a further principle requiring rational agents to update in the manner they predict they will.[25]

5.3 The Principle of Indifference

The previous section discussed various deference principles (the Principal Principle, expert principles, the Reflection Principle) that place additional rational constraints on credence beyond the probability axioms, Ratio Formula, and Conditionalization. Yet each of those deference principles works with a particular kind of evidence—evidence about the chances, about an expert's credences, or about future attitudes. When an agent lacks these specific sorts of evidence about a proposition she's considering, the deference principles will do little to constrain her credences. If an Objective Bayesian (in the normative sense) wants to narrow what's rationally permissible to a single hypothetical prior, he is going to need a stronger principle than these.

The Principle of Indifference is often marketed to do the trick. This is John Maynard Keynes's name for what used to be known as the "principle of insufficient reason":

> The Principle of Indifference asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an *equal* probability. (Keynes 1921, p. 42, emphasis in original)

Applied to degrees of belief, the **Principle of Indifference** holds that if an agent has no evidence favoring any proposition in a partition over any other, she should spread her credence equally over the members of the partition. If I tell you I have painted my house one of the seven colors of the rainbow but tell you nothing more about my selection, the Principle of Indifference requires you to assign credence 1/7 that my house is now violet.

The Principle of Indifference looks like it could settle all open questions about rational credence. An agent could assign specific credences as dictated by portions of her evidence (say, evidence that engages one of the deference principles), then use the Principle of Indifference to settle all remaining questions about her distribution. For example, suppose I tell you that I flipped a fair coin to decide on a house color—heads meant gray, while tails meant a color of the rainbow. You could follow the Principal Principle and assign credence 1/2 to my house's being gray, then follow the Principle of Indifference to distribute the remaining 1/2 credence equally among each of the rainbow colors (so each would receive credence 1/14). This plan seems to dictate a unique rational credence for every proposition in every evidential situation, thereby specifying a unique hypothetical prior distribution.

Unfortunately, the Principle of Indifference has a serious flaw, which was noted by Keynes (among others).[26] Suppose I tell you only that I painted my house some color—I don't tell you what palette I chose from—and you wonder whether it was violet. You might partition the possibilities into the proposition that I painted the house violet and the proposition that I didn't. In that case, the Principle of Indifference will require you to assign credence 1/2 that the house is violet. But if you use the seven colors of the rainbow as your partition, you will assign 1/7 credence that my house is now violet. And if you use the colors in a box of crayons.... The trouble is that faced with the same evidential situation and same proposition to be evaluated, the

Principle of Indifference will recommend different credences depending on which partition you consider.

Might one partition be superior to all the others, perhaps on grounds of the naturalness with which it divides the space of possibilities? (The selection of colors in a crayon box is pretty arbitrary!) Well, consider this example: I just drove eighty miles to visit you. I tell you it took between two and four hours to make the trip, and ask how confident you are that it took less than three. Three hours seems to neatly divide the possibilities in half, so by the Principle of Indifference you assign credence 1/2. Then I tell you I maintained a constant speed throughout the drive, and that speed was between 20 and 40 miles per hour. You consider the proposition that I drove faster than 30mph, and since that evenly divides the possible speeds the Indifference Principle again recommends a credence of 1/2. But these two credence assignments conflict. I drove over 30mph just in case it took me less than two hours and forty minutes to make the trip. So are you 1/2 confident that it took me less than three hours, or that it took me less than two hours forty minutes? If you assign any positive credence that my travel time fell between those durations, the two answers are inconsistent. So once more we need a specified partition (time or velocity) to apply the Principle of Indifference against. But here the decision can't be made on grounds of naturalness: thinking about one's speed of travel is neither more nor less natural than thinking about how long the trip took.[27]

This example is different from the painting example, in that time and velocity require us to consider continuous ranges of possibilities. Infinite possibility spaces introduce a number of complexities we will discuss in the next section, but hopefully the intuitive difficulty is clear. Joseph Bertrand (1888/1972) produced a number of infinite-possibility paradoxes for principles like Indifference. His most famous puzzle (now usually called **Bertrand's Paradox**) asks how probable it is that a chord of a circle will be longer than the side of an inscribed equilateral triangle. Indifference reasoning yields conflicting answers depending on how one specifies the chord in question—by specifying its endpoints, by specifying its orientation and length, by specifying its midpoint, etc.

Since Keynes's discussion, a number of authors have modified his Indifference Principle. Chapter 6 will look in detail at Carnap's proposal. Another well-known suggestion is E.T. Jaynes's (1957a,b) **Maximum Entropy Principle**. Given a partition of the space of possibilities, and a set of constraints on allowable credence distributions over that partition, the Maximum Entropy Principle selects the allowable distribution with the highest entropy. If the

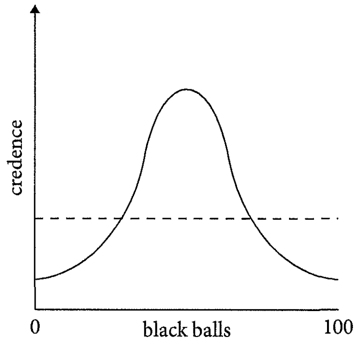

Figure 5.3 Possible urn distributions

partition is finite, containing the propositions $\{Q_1, Q_2, \ldots, Q_n\}$, the entropy of a distribution is calculated as

$$-\sum_{i=1}^{n} \mathrm{cr}(Q_i) \cdot \log \mathrm{cr}(Q_i) \tag{5.13}$$

The technical details of Jaynes's proposal are beyond the level of this book. The intuitive idea, though, is that by *maximizing* entropy in a distribution we *minimize* information.

To illustrate, suppose you know an urn contains 100 balls, each of which is either black or white. Initially, you assign an equal credence to each available hypothesis about how many black balls are in the urn. This "flat" distribution over the urn hypotheses is reflected by the dashed line in Figure 5.3. Then I tell you that the balls were created by a process that tends to produce roughly as many white balls as black. This moves you to the more "peaked" distribution of Figure 5.3's solid curve. The peaked distribution reflects the fact that at the later time you have more information about the contents of the urn. There are various mathematical ways to measure the informational content of a distribution, and it turns out that a distribution's entropy goes down as its information content goes up. So in Figure 5.3, the flat (dashed) distribution has a higher entropy than the peaked (solid) distribution.

Maximizing entropy is thus a strategy for selecting the lowest-information distribution consistent with what we already know. Jaynes's principle says that within the bounds imposed by your evidence, you should select the "flattest" credence distribution available. In a sense, this is a directive not to make any assumptions beyond what you know. As van Fraassen puts it, "one should not jump to unwarranted conclusions, or add capricious assumptions, when accommodating one's belief state to the deliverances of experience" (1981, p. 376). If *all* your evidence about my urn is that it contains 100 black or white balls, it would be strange for you to peak your credences around any particular number of black balls. What in your evidence would justify such a maneuver? The flat distribution seems the most rational option available.[28]

The Maximum Entropy approach has a number of advantages. First, it can easily be extended from finite partitions to infinite partitions by replacing the summation in Equation (5.13) with an integral (and making a few further adjustments). Second, for cases in which an agent's evidence simply delineates a space of doxastic possibilities (without favoring some of those possibilities over others), the Principle of Maximum Entropy yields the same results as the Principle of Indifference. But Maximum Entropy also handles cases involving more complicated sorts of information. Besides restricting the set of possibilities, an agent's evidence might require her credence in one possibility to be twice that in another, or might require a particular conditional credence value for some ordered pair of propositions. No matter the constraints, Maximum Entropy chooses the "flattest" (most entropic) distribution consistent with those constraints. Third, probability distributions selected by the Maximum Entropy Principle have been highly useful in various scientific applications, ranging from statistical mechanics to CT scans to natural language processing.

Yet the Maximum Entropy Principle also has flaws. It suffers from a version of the Indifference Principle's partitioning problem. Maximum Entropy requires us to first select a partition, then accept the most entropic distribution over that partition. But the probability value assigned to a particular proposition by this process often depends on what other propositions appear in the partition. Also, in some evidential situations satisfying the Maximum Entropy Principle both before and after an update requires agents to violate Conditionalization. You can learn more about these problems by studying this chapter's Further Reading.

5.4 Credences for infinitely many possibilities

Suppose I tell you a positive integer was just selected by some process, and tell you nothing more about that process. You need to distribute your credence across all the possible integers that might have been selected. Let's further suppose that you want to assign each positive integer the same credence. In the last section we asked whether, given your scant evidence in this case about the selection process, such an assignment is obligatory—whether you're rationally *required* to assign each positive integer an equal credence. In this section I want to set aside the question of whether an equal distribution is required, and ask whether it's even *possible*.

We're going to have a small, technical problem here with the propositional language over which your credence distribution is assigned. In Chapter 2 we set up propositional languages with a *finite* number of atomic propositions, while a distribution over every positive integer requires infinitely many atomic propositions. Yet there are standard logical methods for dealing with languages containing infinitely many atomic propositions, and even for representing them using a finite number of symbols. For example, we could use "1" to represent the atomic proposition that the number one was selected, "2" to represent two's being selected, "12" to represent twelve's being selected, etc. This will allow us to represent infinitely many atomic propositions with only the standard ten Arabic digits.

So the language isn't the real problem; the real problem is what single credence value you could possibly assign to each and every one of those positive integers. To start seeing the problem, imagine you pick some positive real number r and assign it as your unconditional credence in each positive integer's being picked. For any positive real r you pick, there exists an integer n such that $r > 1/n$. Select such an n, and consider the proposition that the positive integer selected was less than or equal to n. By Finite Additivity (Extended),

$$\text{cr}(1 \vee 2 \vee \ldots \vee n) = \text{cr}(1) + \text{cr}(2) + \ldots + \text{cr}(n) \tag{5.14}$$

Each of the credences on the right-hand side equals r, so your credence in the disjunction is $r \cdot n$. But we selected n such that $r > 1/n$, so $r \cdot n > 1$. Which means the credence on the left-hand side of this equation is greater than 1, and you've violated the probability axioms.

This argument rules out assigning the same positive real credence to each and every positive integer. What other options are there? Historically the most popular proposal has been to assign each positive integer a credence of 0. Yet this proposal creates its own problems.

The first problem with assigning each integer zero credence is that we must reconceive what an unconditional credence of 0 means. So far in this book we have equated assigning credence 0 to a proposition with ruling that proposition out as a live possibility. In this case, though, we've proposed assigning credence 0 to each positive integer while still treating each as a live possibility. So while we will still assign credence 0 to propositions that have been ruled out, there will now be other types of propositions that receive credence 0 as well. Similarly, we may assign credence 1 to propositions of which we are not certain.

Among other things, this reconception of credence 0 will undermine arguments for the Regularity Principle. As stated (Section 4.2), Regularity forbids assigning credence 0 to any logically contingent proposition. The argument there was that one should never entirely rule out a proposition that's logically possible, so one should never assign such a proposition 0 credence. Now we've opened up the possibility of assigning credence 0 to a proposition without ruling it out. So while we can endorse the idea that no contingent proposition should be ruled out, Regularity no longer follows. Moreover, the current proposal provides infinitely many explicit counterexamples to Regularity: we have proposed assigning credence 0 to the contingent proposition that the positive integer selected was one, to the proposition that the integer was two, that it was three, etc.

Once we've decided to think about credence 0 in this new way, we encounter a second problem: the Ratio Formula. In Section 3.1.1 I framed the Ratio Formula as follows:

Ratio Formula: For any P and Q in $\mathcal{L}$, if $\text{cr}(Q) > 0$ then

$$\text{cr}(P \mid Q) = \frac{\text{cr}(P \,\&\, Q)}{\text{cr}(Q)}$$

This constraint relates an agent's conditional credence $\text{cr}(P \mid Q)$ to her unconditional credences *only when* $\text{cr}(Q) > 0$. As stated, it remains silent on how an agent's conditional and unconditional credences relate when $\text{cr}(Q) = 0$.

Yet we surely want to have some rational constraints on that relation for cases in which an agent assigns credence 0 to a contingent proposition that she hasn't ruled out.[29] For example, in the positive integer case consider your conditional credence $\text{cr}(2 \mid 2)$. Surely this conditional credence should equal 1. Yet because the current proposal sets $\text{cr}(2) = 0$, the Ratio Formula cannot tell us anything about $\text{cr}(2 \mid 2)$. And since we've derived all of our rational constraints on conditional credence from the Ratio Formula, the Bayesian system we've set up isn't going to deliver a requirement that $\text{cr}(2 \mid 2) = 1$.[30]

There are various ways to respond to this problem. One interesting suggestion is to reverse the order in which we proceeded with conditional and unconditional credences: We began by laying down fairly substantive constraints (Kolmogorov's probability axioms) on *un*conditional credences, then tied conditional credences to those via the Ratio Formula. On the reverse approach, substantive constraints are first placed on conditional credences, then some further rule relates unconditional to conditional. The simplest such rule is that for any proposition P, $\text{cr}(P) = \text{cr}(P \mid \top)$.

Some advocates of this approach describe it as making conditional credence "basic", but we should be careful not to read too much into debates about what's basic. The way I've approached conditional and unconditional credences in this book, neither is more fundamental than the other in any sense significant to metaphysics or the philosophy of mind. Each is an independently existing type of doxastic attitude, and any rules we offer relating them are strictly *normative* constraints. The only sense in which our unconditionals-first approach has made unconditional credences prior to conditionals is in its order of normative explanation. The Ratio Formula helped us transform constraints on unconditional credences into constraints on conditional credences (as in Section 3.1.2). On the conditionals-first approach, the rule that $\text{cr}(P) = \text{cr}(P \mid \top)$ transforms constraints on conditionals into constraints on unconditionals.

Examples of the conditionals-first approach include Hosiasson-Lindenbaum (1940), Popper (1955), Renyi (1970), and Roeper and Leblanc (1999).[31] Like many of these, Popper's axiom system entails that $\text{cr}(Q \mid Q) = 1$ for any Q that the agent deems possible, regardless of its unconditional credence value. This ensures that $\text{cr}(2 \mid 2) = 1$.

The final problem I want to address with assigning each positive integer 0 unconditional credence of being selected has to do with your unconditional credence that any integer was selected at all. The proposition that some integer was selected is equivalent to the disjunction of the proposition that one was selected, the proposition that two was selected, the proposition that three was selected, etc. Finite Additivity directly governs unconditional credences in disjunctions of two (mutually exclusive) disjuncts; iterating that rule gives us Finite Additivity (Extended), which applies to disjunctions of finitely many disjuncts. But this case concerns an *infinite* disjunction, and none of the constraints we've seen so far relates the unconditional credence of an infinite disjunction to the credences of its disjuncts.

It might seem natural to supplement our credence constraints with the following:

Countable Additivity: For any countable set $\{Q_1, Q_2, Q_3, \ldots\}$ of mutually exclusive propositions in $\mathcal{L}$,

$$\text{cr}(Q_1 \vee Q_2 \vee Q_3 \vee \ldots) = \text{cr}(Q_1) + \text{cr}(Q_2) + \text{cr}(Q_3) + \ldots$$

Notice that Countable Additivity does not apply to sets of *every* infinite size; it applies only to sets of *countably many* members. The set of positive integers is countable, while the set of real numbers is not. (If you are unfamiliar with the differing sizes of infinite sets, I would suggest studying the brief explanation referenced in this chapter's Further Reading.)

Countable Additivity naturally extends the idea behind Finite Additivity to sets of (countably) infinite size. Many authors have found it attractive. Yet in our example it rules out assigning credence 0 to each proposition stating that a particular positive integer was selected. Taken together, the proposition that one was selected, the proposition that two was selected, the proposition that three was selected, etc. form a countable set of mutually exclusive propositions (playing the role of Q_1, Q_2, Q_3, etc. in Countable Additivity). Countable Additivity therefore requires your credence in the disjunction of these propositions to equal the sum of your credences in the individual disjuncts. Yet the latter credences are each 0, while your credence in their disjunction (namely, the proposition that *some* positive integer was selected) should be 1.

So perhaps Countable Additivity wasn't such a good idea after all. The trouble is, without Countable Additivity we lose a very desirable property:

Conglomerability: For each proposition P and partition $\{Q_1, Q_2, Q_3, \dots\}$ in $\mathcal{L}$, $\text{cr}(P)$ is no greater than the largest $\text{cr}(P \mid Q_i)$ and no less than the least $\text{cr}(P \mid Q_i)$.

In other words, if Conglomerability holds then the largest $\text{cr}(P \mid Q_i)$ and the smallest $\text{cr}(P \mid Q_i)$ provide bounds between which $\text{cr}(P)$ must fall.

In defining Conglomerability I didn't say how large the Q-partitions in question are allowed to be. We might think of breaking up the general Conglomerability principle into a number of sub-cases: Finite Conglomerability applies to finite partitions, Countable Conglomerability applies to countable partitions, Continuous Conglomerability applies to partitions of continuum-many members, etc. Finite Conglomerability is guaranteed by the standard probability axioms. You'll prove this in Exercise 5.6, but the basic idea is that by the Law of Total Probability $\text{cr}(P)$ must be a weighted average of the various $\text{cr}(P \mid Q_i)$, so it can't be greater than the largest of them or less than the smallest. With the standard axioms in place, Countable Conglomerability then stands or falls with our decision about Countable Additivity; without Countable Additivity, Countable Conglomerability is false.[32]

We've already seen that the strategy of assigning 0 credence to each positive integer's being selected violates Countable Additivity; let's see how it violates

(Countable) Conglomerability as well.[33] Begin with the following definition: For any positive integer n that's not a multiple of 10, define the n-set as the set of all positive integers that start with n, followed by some number (perhaps 0) of zeroes. So the 1-set is $\{1, 10, 100, 1000, \ldots\}$; the 11-set is $\{11, 110, 1100, 11000, \ldots\}$; the 36-set is $\{36, 360, 3600, 36000, \ldots\}$; etc. Now take the proposition that the integer selected was a member of the 1-set, and the proposition that the integer selected was a member of the 2-set, and the proposition that the integer selected was a member of the 3-set, etc. (Though don't include any ns that are multiples of 10.) The set of these propositions forms a partition. (If you think about it carefully, you'll see that any positive integer that might have been selected belongs to exactly one of these sets.)

The distribution strategy we're considering is going to want to assign

$$\text{cr}(\text{the selected integer is not a multiple of } 10 \mid \text{the selected integer is a member of the 1-set}) = 0 \tag{5.15}$$

Why is that? Well, the only number in the 1-set that is not a multiple of 10 is the number one. The 1-set contains infinitely many positive integers; on the supposition that one of those integers was selected you want to assign equal credence to each one's being selected; so you assign 0 credence to each one's being selected (including the number one) conditional on that supposition. This gives us Equation (5.15). The argument then generalizes; for any n-set you'll have

$$\text{cr}(\text{the selected integer is not a multiple of } 10 \mid \text{the selected integer is a member of that } n\text{-set}) = 0 \tag{5.16}$$

Yet unconditionally it seems rational to have

$$\text{cr}(\text{the selected integer is not a multiple of } 10) = 9/10 \tag{5.17}$$

Conditional on any particular member of our n-set partition, your credence that the selected integer isn't a multiple of 10 is 0. Yet unconditionally, you're highly confident that the integer selected is not a mutiple of 10. This is a flagrant violation of (Countable) Conglomerability—your credences in a particular proposition conditional on each member of a (countable) partition are all the same, yet your unconditional credence in that partition has a very different value!

Why is violating Conglomerability a problem? Well, imagine I'm about to give you some evidence on which you're going to conditionalize. In particular, I'm about to tell you to which of the n-sets the selected integer belongs. Whichever piece of evidence you're about to get, your credence that the integer isn't a multiple of 10 conditional on that evidence is 0. So you can be certain right now that immediately after receiving the evidence—whatever piece of evidence it turns out to be!—your credence that the integer isn't a multiple of 10 will be 0. Yet despite being certain that your better-informed future self will assign a particular proposition a credence of 0, you continue to assign that proposition a credence of 9/10 right now. This is a flagrant violation of the Reflection Principle, as well as generally good principles for attitude management. Our opinions are usually compromises among the pieces of evidence we think we might receive; we expect that some potential future pieces of evidence would change our views in one direction, while others would press in the other. If we know that no matter what evidence comes in we're going to be pulled away from our current opinion in the same direction, it seems irrationally stubborn to maintain our current opinion and not move in that direction right now. Conglomerability embodies these principles of good evidential hygiene; without Conglomerability our interactions with evidence begin to look absurd.

Where does this leave us? We wanted to find a way to assign an equal credence to each positive integer's being selected. We quickly concluded that that equal credence could not be a positive real number. So we considered assigning credence 0 to each integer's being selected. Doing so violates Countable Additivity (a natural extension of our finite principles for calculating credences in disjunctions) and Conglomerability, which looks desirable for a number of reasons. Are there any *other* options?

I will briefly mention two further possibilities. The first possibility is to assign each positive integer an **infinitesimal** credence of having been selected. To work with infinitesimals, we extend the standard real-number system to include numbers that are greater than 0 but smaller than all the positive reals. If we assign each integer an infinitesimal credence of having been picked, we avoid the problems with assigning a positive real and also the problems of assigning 0. (For instance, if you pile enough infinitesimals together they can sum to 1.) Yet infinitesimal numbers have a great deal of mathematical structure, and it's not clear that the extra structure plausibly represents any feature of agents' attitudes.[34] Moreover, the baroque mathematics of infinitesimals introduces troubles of its own (see Further Reading). So perhaps only one viable option remains: Perhaps if you learn a positive integer was just selected, it's *impossible* to assign equal credence to each of the possibilities consistent with what you know.[35]

5.5 Jeffrey Conditionalization

Section 4.1.1 showed that conditionalizing on new evidence creates and retains certainties; evidence gained between two times becomes certain at the later time and remains so ever after. Contraposing, if an agent updates by Conditionalization and gains no certainties between two times, it must be because she gained no evidence between those times. In that section we also saw that if an agent gains no evidence between two times, Conditionalization keeps her credences fixed. Putting all this together, we see that under Conditionalization an agent's credences change just in case she gains new certainties.

As we noted in Section 4.2, mid-twentieth-century epistemologists like C.I. Lewis defended this approach by citing sense data as the foundational evidential certainties. Many contemporary epistemologists are uncomfortable with this kind of foundationalism (and with appeals to sense data in general). Richard C. Jeffrey, however, had a slightly different concern, which he expressed with the following example and analysis:

> The agent inspects a piece of cloth by candlelight, and gets the impression that it is green, although he concedes that it might be blue or even (but very improbably) violet. If G, B, and V are the propositions that the cloth is green, blue, and violet, respectively, then the outcome of the observation might be that, whereas originally his degrees of belief in G, B, and V were .30, .30, and .40, his degrees of belief in those same propositions after the observation are .70, .25, and .05. If there were a proposition E in his preference ranking which described the precise quality of his visual experience in looking at the cloth, one would say that what the agent learned from the observation was that E is true....
>
> But there need be no such proposition E in his preference ranking; nor need any such proposition be expressible in the English language. Thus, the description "The cloth looked green or possibly blue or conceivably violet," would be too vague to convey the precise quality of the experience. Certainly, it would be too vague to support such precise conditional probability ascriptions as those noted above. It seems that the best we can do is to describe, not the quality of the visual experience itself, but rather its effects on the observer, by saying, "After the observation, the agent's degrees of belief in G, B, and V were .70, .25, and .05." (1965, p. 154)

Jeffrey worried that even if we grant the existence of a sense datum for each potential learning experience, the quality of that sense datum might not be representable in a proposition to which the agent could assign certainty, or at least might not be representable in a precise-enough proposition to differentiate that sense datum from other nearby data with different effects on the agent's credences.

At the time Jeffrey was writing, the standard Bayesian updating norm (updating by Conditionalization) relied on the availability of such propositions. So Jeffrey proposed a new updating rule, capable of handling examples like the cloth one above. While he called it **probability kinematics**, it is now universally known as

Jeffrey Conditionalization: Given any t_i and t_j with $i < j$, any A in $\mathcal{L}$, and a finite partition $\{B_1, B_2, \ldots, B_n\}$ in $\mathcal{L}$ whose members each have nonzero cr_i,

$$\mathrm{cr}_j(A) = \mathrm{cr}_i(A \mid B_1) \cdot \mathrm{cr}_j(B_1) + \mathrm{cr}_i(A \mid B_2) \cdot \mathrm{cr}_j(B_2) + \ldots + \mathrm{cr}_i(A \mid B_n) \cdot \mathrm{cr}_j(B_n)$$

Let's apply Jeffrey Conditionalization to the cloth example. Suppose I'm fishing around in a stack of my family's clean laundry hoping to pull out any shirt that belongs to me, but the lighting is dim because I don't want to turn on the overheads and awaken my wife. The color of a shirt in the stack would be a strong clue as to whether it was mine, as reflected by my conditional credences:

$$\begin{aligned} \mathrm{cr}_1(\text{mine} \mid G) &= 0.80 \\ \mathrm{cr}_1(\text{mine} \mid B) &= 0.50 \\ \mathrm{cr}_1(\text{mine} \mid V) &= 0 \end{aligned} \tag{5.18}$$

(For simplicity's sake we imagine green, blue, and violet are the only color shirts I might fish out of the stack.) At t_1 I pull out a shirt. Between t_1 and t_2 I take a glimpse of the shirt. According to Jeffrey's story, my unconditional credence distributions across the $G/B/V$ partition are:

$$\begin{array}{lll} \mathrm{cr}_1(G) = 0.30 & \mathrm{cr}_1(B) = 0.30 & \mathrm{cr}_1(V) = 0.40 \\ \mathrm{cr}_2(G) = 0.70 & \mathrm{cr}_2(B) = 0.25 & \mathrm{cr}_2(V) = 0.05 \end{array} \tag{5.19}$$

Applying Jeffrey Conditionalization, I find my credence in the target proposition at the later time by combining my post-update unconditional credences across the partition with my pre-update credences in the target proposition conditional on members of the partition. This yields:

$$\begin{aligned}
&\text{cr}_2(\text{mine}) = \\
&\text{cr}_1(\text{mine} \mid G) \cdot \text{cr}_2(G) + \text{cr}_1(\text{mine} \mid B) \cdot \text{cr}_2(B) + \text{cr}_1(\text{mine} \mid V) \cdot \text{cr}_2(V) = \\
&0.80 \cdot 0.70 + 0.50 \cdot 0.25 + 0 \cdot 0.05 = \\
&0.685
\end{aligned} \tag{5.20}$$

At t_2 I'm fairly confident that the shirt I've selected is mine. How confident was I at t_1, before I caught my low-light glimpse? A quick calculation with the Law of Total Probability reveals that $\text{cr}_1(\text{mine}) = 0.39$. But it's more interesting to see what happens when we apply the Law of Total Probability to my credences at t_2:

$$\begin{aligned}
&\text{cr}_2(\text{mine}) = \\
&\text{cr}_2(\text{mine} \mid G) \cdot \text{cr}_2(G) + \text{cr}_2(\text{mine} \mid B) \cdot \text{cr}_2(B) + \text{cr}_2(\text{mine} \mid V) \cdot \text{cr}_2(V)
\end{aligned} \tag{5.21}$$

Take a moment to compare Equation (5.21) with the first two lines of Equation (5.20). Equation (5.21) expresses a feature that my t_2 credence distribution must have if it is to satisfy the probability axioms and Ratio Formula. Equation (5.20) tells me how to set my t_2 credences by Jeffrey Conditionalization. The only way to make these two equations match—the only way to square the Jeffrey update with the probability calculus—is if $\text{cr}_1(\text{mine} \mid G) = \text{cr}_2(\text{mine} \mid G)$, $\text{cr}_1(\text{mine} \mid B) = \text{cr}_2(\text{mine} \mid B)$, etc.

Why should these conditional credences stay constant over time? Well, at any given time my credence that the shirt I've selected is mine is a function of two kinds of credences: first, my unconditional credence that the shirt is a particular color; and second, my conditional credence that the shirt is mine given that it's a particular color. When I catch a glimpse of the shirt between t_1 and t_2, *only the first kind of credence changes.* I change my opinion about what color the shirt is, but I don't change my confidence that it's my shirt given that (say) it's green. Throughout the example I have a fixed opinion about what percentage of the green shirts in the house are mine; I simply gain information about whether *this* shirt is green. So while my unconditional color credences change, my credences conditional on the colors remain.

This discussion reveals a general feature of Jeffrey Conditionalization. You'll prove in Exercise 5.8 that an agent's credences between two times update by Jeffrey Conditionalzation just in case the following condition obtains:

Rigidity: For any A in $\mathcal{L}$ and any B_m in $\{B_1, B_2, \ldots, B_n\}$,

$$\mathrm{cr}_j(A \mid B_m) = \mathrm{cr}_i(A \mid B_m)$$

So Jeffrey Conditionalization using a particular partition $\{B_1, B_2, \ldots, B_n\}$ is appropriate only when the agent's credences conditional on the B_m remain constant across two times. Jeffrey thought this was reasonable for updates that "originate" in the B_m partition.[36] In the cloth example, all my credence changes between t_1 and t_2 are *driven* by the changes in my color credences caused by my experience. So if I tell you my credences at t_1, and then tell you my unconditional credences in the color propositions at t_2, this should suffice for you to work out the rest of my opinions at t_2. Jeffrey Conditionalization makes that possible.

Rigidity can help us perform Jeffrey Conditionalization updates on a probability table. Given the partition $\{B_1, B_2, \ldots, B_n\}$ in which an update originates, we divide the lines of the table into "blocks": the B_1 block contains all the lines consistent with B_1; the B_2 block contains all the lines consistent with B_2; etc. The agent's experience between times t_i and t_j directly sets her unconditional cr_j-values for the B_m; in other words, it tells us what each block must sum to at t_j. Once we know a block's cr_j total, we set the values on individual lines within that block by keeping them in the same proportions as at t_i. (This follows from Rigidity's requirement that each line have the same cr_2-value conditional on a given B_m as it did at t_i.) That is, we multiply all the cr_i-values in a block by the *same* constant so that their cr_j-values achieve the appropriate sum.

Figure 5.4 shows this process for the colored shirt example. I've built the table around a simplified partition of doxastic possibilities in the problem, but I could've made a probability table with the full list of state-descriptions and everything would proceed the same way. I calculated the cr_1-values in the table from Equations (5.18) and (5.19). How do we then derive the credences at t_2?

The credence change between t_1 and t_2 originates in the $G/B/V$ partition. So the "blocks" on this table will be pairs of adjacent lines: the first pair of lines (on which G is true), the second pair of lines (B lines), and the third pair of

partition member	cr_1	cr_2
G & mine	0.24	0.56
G & ~mine	0.06	0.14
B & mine	0.15	0.125
B & ~mine	0.15	0.125
V & mine	0	0
V & ~mine	0.40	0.05

Figure 5.4 Jeffrey Conditionalization across a partition

V lines. Let's work with the B-block first. In Jeffrey's story, glimpsing the shirt sends me to $cr_2(B) = 0.25$. So on the table, the third and fourth lines must have cr_2-values summing to 0.25. At t_1 these lines were in a $1 : 1$ ratio, so they must maintain that ratio at t_2. This leads to cr_2-values of 0.125 on both lines. Applying a similar process to the G- and V-blocks yields the remaining cr_2-values.

Once you understand this block-updating process, you can see that traditional updating by Conditionalization is a special case of updating by Jeffrey Conditionalization. When you update by Conditionalization on some evidential proposition E, your probability table divides into two blocks: lines consistent with E versus $\sim E$ lines. After the update, the $\sim E$ lines go to zero, while the E lines are multiplied by a constant so that they sum to 1.

This tells us how Jeffrey Conditionalization relates to traditional (or "strict") Conditionalization mathematically. But how should we understand their relation philosophically? Suppose we class learning experiences into two kinds: those that send some proposition to certainty and those that don't. Jeffrey Conditionalization seems to be a universal updating rule, applying to both kinds of experience. When experience does send a proposition to certainty, Jeffrey Conditionalization provides the same advice as strict Conditionalization. But Jeffrey Conditionalization also provides guidance for learning experiences of the second kind.

Now the defender of Regularity (the principle forbidding extreme unconditional credence in logically contingent propositions) will maintain that *only* the second kind of learning experience ever occurs (at least to rational agents), and therefore that strict Conditionalization should *never* be applied in practice. All experience ever does is shuffle an agent's unconditional credences over some partition, without sending any partition members to extremity. Jeffrey Conditionalization tells us how such changes over a partition affect the rest of the agent's credence distribution.

But one can identify an important role for Jeffrey Conditionalization even without endorsing Regularity. To establish the need for his new kinematics, Jeffrey only had to argue that *some* experiences of the second kind exist—*sometimes* we learn without gaining certainties. In that case we need a more general updating rule than strict Conditionalization, and Jeffrey Conditionalization provides one.

Yet despite being such a flexible tool, Jeffrey Conditionalization has its drawbacks. For instance, while applications of strict Conditionalization are always commutative, Jeffrey updates that do not send propositions to certainty may not be. The simplest example of this phenomenon (which Jeffrey readily acknowledged) occurs when one learning experience sends some B_m in the partition to unconditional credence p, while the next experience sends that same partition member to a different credence value q. Applying Jeffrey Conditionalization to the experiences in that order will leave the agent with a final unconditional credence in B_m of q, while applying Jeffrey's rule to the same experiences in the opposite order will result in a final B_m credence of p. This commutativity failure is problematic if you think that the effects of evidence on an agent should not depend on the order in which pieces of evidence arrive.[37]

Finally, Jeffrey Conditionalization may not provide a recipe for *every* type of learning experience. Traditional Conditionalization covers experiences that set unconditional credences to certainty. Jeffrey Conditionalization generalizes to experiences that set unconditional credences to nonextreme values. But what if an experience affects an agent by directly altering her *conditional* credences? How can we calculate the effects of such an experience on her other degrees of belief? Readers interested in that question might begin by exploring van Fraassen's "Judy Benjamin Problem" (1981), an example in which direct alteration of conditional credences plausibly occurs, but which cannot be analyzed using Jeffrey Conditionalization.[38]

5.6 Exercises

Unless otherwise noted, you should assume when completing these exercises that the credence distributions under discussion satisfy the probability axioms and Ratio Formula. You may also assume that whenever a conditional credence expression occurs, the needed proposition has nonzero unconditional credence so that conditional credences are well defined.

Problem 5.1. At noon I rolled a six-sided die. It came from either the Fair Factory (which produces exclusively fair dice), the Snake-Eyes Factory (which produces dice with a 1/2 chance of coming up one and equal chance of each other outcome), or the Boxcar Factory (which produces dice with a 1/4 chance of coming up six and equal chance of each other outcome).

(a) Suppose you use the Principle of Indifference to assign equal credence to each of the three factories from which the die might have come. Applying the Principal Principle, what is your credence that my die roll came up three?

(b) Maria tells you that the die I rolled didn't come from the Boxcar Factory. If you update on this new evidence by Conditionalization, how confident are you that the roll came up three?

(c) Is Maria's evidence admissible with respect to the outcome of the die roll? Explain.

(d) After you've incorporated Maria's information into your credence distribution, Ron tells you the roll didn't come up six. How confident are you in a three after conditionalizing on Ron's information?

(e) Is Ron's evidence admissible with respect to the outcome of the die roll? Explain.

Problem 5.2. The expert deference principle in Equation (5.9) resembles the Principal Principle in many ways. Yet the expert deference principle makes no allowance for anything like inadmissible information. What kind of information should play the role for expert deference that inadmissible information plays for deference to chances? How should Equation (5.9) be modified to take such information into account?

Problem 5.3. Suppose t_1, t_2, and t_3 are three times, with the indices reflecting their temporal order. At t_1, you satisfy the probability axioms, Ratio Formula, and Reflection Principle. You are also certain at t_1 that you will satisfy these constraints at t_2. However, for some proposition X your t_1 credences are equally divided between the following two (mutually exclusive and exhaustive) hypotheses about what your t_2 self will think of your t_3 credences:

$$\textbf{Y:}\quad (\mathrm{cr}_2[\mathrm{cr}_3(X) = 1/10] = 1/3) \;\&\; (\mathrm{cr}_2[\mathrm{cr}_3(X) = 2/5] = 2/3)$$
$$\textbf{Z:}\quad (\mathrm{cr}_2[\mathrm{cr}_3(X) = 3/8] = 3/4) \;\&\; (\mathrm{cr}_2[\mathrm{cr}_3(X) = 7/8] = 1/4)$$

Given all this information, what is $\mathrm{cr}_1(X)$? (Be sure to explain your reasoning clearly.)

Problem 5.4. Can you think of any kind of real-world situation in which it would be rationally permissible to violate the Reflection Principle? Explain the situation you're thinking of, and why it would make a Reflection violation okay.

Problem 5.5. Jingyi assigns the t_1 credences indicated by the probability table below. Then between t_1 and t_2, she learns $P \supset Q$.

P	Q	cr_1
T	T	0.4
T	F	0.2
F	T	0.2
F	F	0.2

(a) Determine Jingyi's credence distribution at t_2. Then use Equation (5.13) to calculate the entropy of both cr_1 and cr_2 over the partition containing the four P/Q state-descriptions.[39]
(b) Use the concept of information content to explain why the entropy of Jingyi's distribution changed in the direction it did between t_1 and t_2.
(c) Create a probabilistic credence distribution that assigns the same unconditional value to P as cr_1, but has a higher entropy over the P/Q state-description partition.
(d) Use the partition containing just P and $\sim P$ to calculate the entropy for cr_1 and for your distribution from part (c). What does this tell you about the partition-dependence of entropy comparisons?

Problem 5.6. Using Non-Negativity, Normality, Finite Additivity, the Ratio Formula, and any results we've proven from those four, prove Finite Conglomerability. (Hint: The Law of Total Probability may be useful here.)

Problem 5.7. Suppose that at t_1 you assign a "flat" credence distribution over language $\mathcal{L}$ whose only two atomic propositions are B and C—that is, you assign equal credence to each of the four state-descriptions of $\mathcal{L}$. Between t_1 and t_2 you perform a Jeffrey Conditionalization that originates in the $B/{\sim}B$ partition and sets $\text{cr}_2(B) = 2/3$. Between t_2 and t_3 you perform a Jeffrey Conditionalization that originates in the $C/{\sim}C$ partition and sets $\text{cr}_3(C) = 3/4$.

(a) Calculate your cr_2 and cr_3 distributions.
(b) Does your credence in B change between t_2 and t_3? Does your credence in C change between t_1 and t_2?

(c) By talking about probabilistic independence at t_1 and t_2, explain the changes or lack of changes you observed in parts (b) and (c).

(d) Now start again with the flat t_1 distribution, but apply the Jeffrey Conditionalizations in the opposite order. (First an update that sets the C credence to 3/4, then an update that sets B to 2/3.)

(e) Is the cr_3 distribution you obtained in part (e) the same as the one from part (a)? Does this always happen when you reverse the order of Jeffrey Conditionalizations? If not, why do you think it happened in this case?

Problem 5.8. Prove that Jeffrey Conditionalization is equivalent to Rigidity. That is: Given any times t_i and t_j, and any finite partition $\{B_1, B_2, \ldots, B_n\}$ in $\mathcal{L}$ whose members each have nonzero cr_i, the following two conditions are equivalent:

1. For all A in $\mathcal{L}$, $\text{cr}_j(A) = \text{cr}_i(A \mid B_1) \cdot \text{cr}_j(B_1) + \text{cr}_i(A \mid B_2) \cdot \text{cr}_j(B_2) + \ldots + \text{cr}_i(A \mid B_n) \cdot \text{cr}_j(B_n)$.
2. For all A in $\mathcal{L}$ and all B_m in the partition, $\text{cr}_j(A \mid B_m) = \text{cr}_i(A \mid B_m)$.

(Hint: Complete two proofs—first condition 2 from condition 1, then vice versa.)

Problem 5.9. Suppose we apply Jeffrey Conditionalization over a finite partition $\{B_1, B_2, \ldots, B_n\}$ in $\mathcal{L}$ to generate cr_2 from cr_1. Show that we could have obtained the same cr_2 from cr_1 in the following way: start with cr_1; Jeffrey Conditionalize it in a particular way over a partition containing only two propositions; Jeffrey Conditionalize the result of *that* operation in a particular way over a partition containing only two propositions (possibly different from the ones used the first time); repeat this process a finite number of times until cr_2 is eventually obtained.[40]

5.7 Further reading

Subjective and Objective Bayesianism

Maria Carla Galavotti (2005). *Philosophical Introduction to Probability*. CSLI Lecture Notes 167. Stanford, CA: CSLI Publications

Excellent historical introduction to the many ways "probability" has been understood by the philosophical and statistical community.

Alan Hájek (2019). Interpretations of Probability. In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Fall 2019. URL: http://plato.stanford.edu/archives/fall2019/entries/probability-interpret/

Survey of the various interpretations of probability, with extensive references.

Bruno de Finetti (1931/1989). Probabilism: A Critical Essay on the Theory of Probability and the Value of Science. *Erkenntnis* 31, pp. 169–223. (Translation of B. de Finetti, *Probabilismo*, Logos 14: 163–219)

Classic paper critiquing objective interpretations of probability and advocating a Subjective Bayesian (in the semantic sense) approach.

Donald Gillies (2000). Varieties of Propensity. *British Journal for the Philosophy of Science* 51, pp. 807–35

Reviews different versions of the propensity theory and their motivations. Focuses at the end on how propensity theories might respond to Humphreys's Paradox.

DEFERENCE PRINCIPLES

David Lewis (1980). A Subjectivist's Guide to Objective Chance. In: *Studies in Inductive Logic and Probability*. Ed. by Richard C. Jeffrey. Vol. 2. Berkeley: University of California Press, pp. 263–94

Lewis's classic article laying out the Principal Principle and its consequences for theories of credence and chance.

Adam Elga (2007). Reflection and Disagreement. *Noûs* 41, pp. 478–502

Offers principles for deferring to many different kinds of agents, including experts, gurus (individuals with good judgment who lack some of your evidence), past and future selves, and peers (whose judgment is roughly as good as your own).

Bas C. van Fraassen (1984). Belief and the Will. *The Journal of Philosophy* 81, pp. 235–56

Article in which van Fraassen proposes and defends the Reflection Principle.

Jonathan Weisberg (2007). Conditionalization, Reflection, and Self-Knowledge. *Philosophical Studies* 135, pp. 179–97

Discusses conditions under which Reflection can be derived from Conditionalization, and vice versa.

Richard Pettigrew and Michael G. Titelbaum (2014). Deference Done Right. *Philosophers' Imprint* 14, pp. 1–19

Attempts to get the formulation of deference principles precisely right, including expert deference principles, the Reflection Principle, and principles for higher-order credences. Particularly concerned with making those principles consistent with Conditionalization and with the possibility of ignorance about what's rationally required.

The Principle of Indifference

John Maynard Keynes (1921). *Treatise on Probability*. London: Macmillan and Co., Limited

Chapter IV contains Keynes's famous discussion of the Principle of Indifference.

E. T. Jaynes (1957a). Information Theory and Statistical Mechanics I. *Physical Review* 106, pp. 620–30

E. T. Jaynes (1957b). Information Theory and Statistical Mechanics II. *Physical Review* 108, pp. 171–90

E.T. Jaynes introduces the Maximum Entropy approach.

Colin Howson and Peter Urbach (2006). *Scientific Reasoning: The Bayesian Approach*. 3rd edition. Chicago: Open Court

Section 9.a covers the Indifference Principle, Harold Jeffreys's attempts to make it partition-invariant, and then Jaynes's Maximum Entropy theory. Very clear on the flaws of all of these approaches.

Teddy Seidenfeld (1986). Entropy and Uncertainty. *Philosophy of Science* 53, pp. 467–91

A general discussion of the flaws with Jaynes's Maximum Entropy approach; especially good on its incompatibility with Bayesian conditionalization. Also contains useful references to Jaynes's many defenses of Maximum Entropy over the years and to the critical discussion that has ensued.

Credences for Infinite Possibilities

David Papineau (2012). *Philosophical Devices: Proofs, Probabilities, Possibilities, and Sets.* Oxford: Oxford University Press

Chapter 2 offers a highly accessible introduction to the cardinalities of various infinite sets. (Note that Papineau uses "denumerable" where we use the term "countable".)

Alan Hájek (2003). What Conditional Probability Could Not Be. *Synthese* 137, pp. 273–323

Assesses the viability of the Ratio Formula as a definition of conditional probability in light of various infinite phenomena and plausible violations of Regularity.

Colin Howson (2014). Finite Additivity, Another Lottery Paradox and Conditionalisation. *Synthese* 191, pp. 989–1012

Neatly surveys arguments for and against Countable Additivity, then argues for dropping Conditionalization as a universal update rule over accepting infinite additivity principles.

Timothy Williamson (2007). How Probable Is an Infinite Sequence of Heads? *Analysis* 67, pp. 173–80

Brief introduction to the use of infinitesimals in probability distributions, followed by an argument against using infinitesimals to deal with infinite cases.

Kenny Easwaran (2014b). Regularity and Hyperreal Credences. *Philosophical Review* 123, pp. 1–41

Excellent, comprehensive discussion of the motivations for Regularity, the mathematics of infinitesimals, arguments against using infinitesimals to secure Regularity (including Williamson's argument), and an alternative approach.

JEFFREY CONDITIONALIZATION

Richard C. Jeffrey (1965). *The Logic of Decision*. 1st edition. McGraw-Hill Series in Probability and Statistics. New York: McGraw-Hill

Chapter 11 contains Jeffrey's classic presentation of his "probability kinematics", now universally known as "Jeffrey Conditionalization".

Notes

1. The frequency theory is sometimes referred to as "frequentism" and its adherents as "frequentists". However "frequentism" more often refers to a school of statistical practice at odds with Bayesianism (which we'll discuss in Chapter 13). The ambiguity probably comes from the fact that most people in that statistical school also adopt the frequency theory as their interpretation of probability. But the positions are logically distinct and should be denoted by different terms. So I will use "frequency theory" here, and reserve "frequentism" for my later discussion of the statistical approach.
2. For many, many more see Hájek (1996) and its sequel Hájek (2009b).
3. The frequency theory will also need to work with counterfactuals if nonextreme probabilities can be meaningfully ascribed to a priori truths, or to metaphysical necessities. (Might a chemist at some point have said, "It's highly probable that water is H_2O"?) Assigning nonextreme frequencies to such propositions' truth involves possible worlds far away from the actual.
4. This difficulty for the propensity theory is often known as **Humphreys's Paradox**, since it was proposed in Humphreys (1985).

   One might respond to Humphreys's Paradox by suggesting that propensities don't follow the standard mathematical rules of probability. And honestly, it's not obvious why they should. The frequency theory clearly yields probabilistic values: in any sequence of event repetitions a given outcome has a non-negative frequency, the tautologous outcome has a frequency of 1, and mutually exclusive outcomes have frequencies summing to the frequency of their disjunction. In fact, Kolmogorov's axioms can be read as a generalization of the mathematics of event frequencies to cases involving irrational and infinite quantities. But establishing that propensity values (or objective chances) satisfy the probability axioms takes *argumentation* from one's metaphysics of propensity. Nevertheless, most authors who work with propensities assume that they satisfy the axioms; if they didn't, the propensity interpretation's probabilities wouldn't count as probabilities in the mathematician's sense (Section 2.2).

5. One could focus here on a metaphysical distinction rather than a semantic one—instead of asking what "probability" talk *means*, I could ask what probabilities *are*. But some of the probability interpretations we will discuss don't have clear metaphysical commitments. The logical interpretation, for instance, takes probability to be a logical relation, but need not go on to specify an ontology for such relations. So I will stick with a semantic distinction, which in any case matches how these questions were discussed in much of twentieth-century analytic philosophy.
6. In the twentieth century Subjective Bayesianism was also typically read as a form of expressivism; an agent's "probability" talk *expressed* her quantitative attitudes toward propositions without having truth-conditions. Nowadays alternative semantics are available that could interpret "probability" talk in a more cognitivist mode while still reading such talk as reflecting subjective degrees of belief (Weatherson and Egan 2011).
7. See Hacking (1971) for discussion of Leibniz's position.
8. Carnap himself did not believe all "probability" talk picked out the logical values just described. Instead, he thought "probability" was ambiguous between two meanings, one of which was logical probability and the other of which had more of a frequency interpretation.
9. There is disagreement about whether the logical and evidential interpretations of probability should be considered Objective Bayesian in the semantic sense. Popper (1957) says that objective interpretations make probability values objectively *testable*. Logical and evidential probabilities don't satisfy that criterion, and Popper seems to class them as subjective interpretations. Yet other authors (such as Galavotti 2005) distinguish between logical and subjective interpretations. I have defined the semantic Subjective/Objective Bayesian distinction so that logical and evidential interpretations count as Objective; while they may be normative for the attitudes of agents, logical and evidential probabilities do not vary with the attitudes particular agents or groups of agents possess.
10. As I explained in Chapter 4, note 17, defining hypothetical priors as regular does not commit us to the Regularity Principle as a rational constraint.
11. Those who believe that "probability" is used in many ways—or that there are many different kinds of entities that count as probabilities—sometimes use the terms "subjective probability" and "objective probability". On this usage, subjective probabilities are agents' credences, while objective probabilities include all the kinds of probabilities we've mentioned that are independent of particular agents' attitudes.
12. To assign H a credence exactly equal to the observed frequency of heads would be to follow what Reichenbach (1938) called the **straight rule**. Interestingly, it's impossible to construct a hypothetical prior satisfying the probability axioms that allows an agent to obey the straight rule in its full generality. However, Laplace (1814/1995) proved that if an agent's prior satisfies the Principle of Indifference (adopting a "flat" distribution somewhat like the dashed line in Figure 5.3), her posteriors will obey the **rule of succession**: after seeing h of n tosses come up heads, her credence in H will be $(h+1)/(n+2)$. As the number of tosses increases, this credence approaches the observed frequency of heads.

    Given these difficulties aligning credences and observed frequencies, anyone who thinks credences should match chances needs to describe a hypothetical prior making such a match possible. In a moment we'll see Lewis doing this with the Principal Principle.

13. Since the ratio of B-outcomes to A-events must always fall between 0 and 1, this principle sheds some light on why credence values are usually scaled from 0 to 1. (Compare note 4 above.)
14. There's also the problem that we sometimes have data from overlapping reference classes applying to the same case, neither of which is a subclass of the other. *The Book of Odds* (Shapiro, Campbell, and Wright 2014, p. 137) reports that 1 in 41.7 adults in the United States aged 20 or older experiences heart failure in a given year. For non-Hispanic white men 20 or older, the number is 1 in 37. But only 1 in 500 men aged 20–39 experiences heart failure in a given year. In setting my credence that I will have a heart attack this year, should I use the data for non-Hispanic white men over 20 or the data for men aged 20–39?
15. Here I'm thinking especially of the following: "What makes it be so that a certain reasonable initial credence function and a certain reasonable system of basic intrinsic values are both yours is that you are disposed to act in more or less the ways that are rationalized by the pair of them together, taking into account the modification of credence by conditionalizing on total evidence" (Lewis 1980, p. 288).
16. My explanation at this point in the text of screening-off in the Principal Principle fits very naturally with a propensity-style account of chance. I'm unsure whether it could be made to work on Lewis's own "best system" theory of chance (Lewis 1994). As far as I know, Lewis himself never explains why the screening-off captured by the Principal Principle should obtain, except to say that it matches our best intuitions about how rational agents assign credences to chance events.
17. The notion of screening off in play here is the one I described in Chapter 3, note 9 for continuous random variables. The objective chance of H is a continuous variable, so facts about $\text{Ch}(H)$ screen off known flip frequencies from H in the sense that conditional on setting $\text{Ch}(H)$ to any particular value, known frequency information becomes irrelevant to H.
18. Notice that the time t_i to which the chance in the Principal Principle is indexed need not be the time at which an agent assigns her credence concerning the experimental outcome A. In our coin example, the agent forms her credence at 1 p.m. about the coin flip outcome at noon using information about the chances *at noon*. This is significant because on some metaphysical theories of chance, once the coin flip lands heads (or tails) the chance of H goes to 1 (or 0) forevermore. Yet even if the chance of H has become extreme by 1 p.m., the Principal Principle may still direct an agent to assign a nonextreme 1 p.m. credence to H if all she knows are the chances from an earlier time. (Getting this last point wrong is the most frequent mistake I see people make in applying the Principal Principle. For more such mistakes, see Meacham 2010b.)

    I should also note that because chances are time-indexed, the notion of admissibility must be time-indexed as well. The information about the wad of chewing gum is admissible relative to 11:30 a.m. chances—learning about the chewing gum affects your credence about the flip outcome by way of your opinions about the 11:30 a.m. chances. But the information that chewing gum was stuck to the coin after 11 a.m. is *in*admissible relative to the 11 a.m. chances. (Chewing gum information affects your credence in H, but not by influencing your opinions about the chances associated with the coin at 11 a.m.) So strictly speaking we should ask whether a piece of information is admissible *for* a particular proposition *relative* to the chances at a given time. I have suppressed this complication in the main text.

19. For a partition containing only two members (call them C_1 and C_2), the unconditional credence form of the Law of Total Probability tells us that

$$\mathrm{cr}(A) = \mathrm{cr}(A \mid C_1) \cdot \mathrm{cr}(C_1) + \mathrm{cr}(A \mid C_2) \cdot \mathrm{cr}(C_2)$$

The conditional credence form (generated by the procedure described in Section 3.1.2) tells us that for any E with $\mathrm{cr}(E) > 0$,

$$\mathrm{cr}(A \mid E) = \mathrm{cr}(A \mid C_1 \mathbin{\&} E) \cdot \mathrm{cr}(C_1 \mid E) + \mathrm{cr}(A \mid C_2 \mathbin{\&} E) \cdot \mathrm{cr}(C_2 \mid E)$$

20. One caveat I *won't* get into is that Lewis's original (1980) formulation of the Principal Principle becomes inconsistent if we allow propositions about chances to have chances of their own, and those chances of chances may be nonextreme. For why we might allow this, and how Lewis (and others) reformulated the Principal Principle in response, see Lewis (1994) and the literature that followed.
21. Equation (5.9) directs the assignment of your unconditional credences only when information about the opinion of a particular expert is your *total* relevant evidence concerning proposition A. If you have additional information about A (perhaps the opinion of a second expert?), the relevant condition in the conditional credence on the left-hand side of Equation (5.9) is no longer just $\mathrm{cr}_E(A) = x$. (See Exercise (5.2) for more on this point.)
22. Supposing that your future credences result from your present credences by conditionalization guarantees that your future self will possess at least as much evidence as your present self. But it also has the advantage of guaranteeing that future and present selves both work from the same hypothetical prior distribution (because of the Hypothetical Priors Theorem, Section 4.3). It's worth thinking about whether an agent should defer to the opinions of a database expert who, while having evidence that's a strict superset of the agent's, analyzes that evidence using different epistemic standards.
23. The justification I've just provided for Equation (5.11) explicitly uses every one of the enumerated conditions except Condition 3. Condition 3 is necessary so that the conditional credence in Equation (5.11) is well defined according to the Ratio Formula.
24. One complication here is that van Fraassen sometimes describes Reflection as relating attitudes, but at other times portrays it as being about various *acts* of commitment, and therefore more directly concerned with assertions and avowals than with particular mental states.
25. The Reflection Principle applies to times t_i and t_j with j strictly greater than i. What would happen if we applied it when $j = i$? In that case we'd have a principle for how an agent's current credences should line up with her credences about her current credences. This principle would engage the results of an agent's introspection to determine what her current credences are. An agent's credences about her own current credences are her **higher-order credences**, and they have been the subject of much Bayesian scrutiny (e.g., Skyrms 1980b). The core issue is how much access a rational agent is required to have to the contents of her own mind.

26. Joyce (2005) reports that this sort of problem was first identified by John Venn in the 1800s.
27. This example is adapted from one in Salmon (1966, pp. 66–7). A related example is van Fraassen's (1989) Cube Factory, which describes a factory making cubes of various sizes and asks how confident I should be that a given manufactured cube has a size falling within a particular range. The Principle of Indifference yields conflicting answers depending on whether cube size is characterized using side length, face area, or volume.
28. In Chapter 14 we will discuss other potential responses to this kind of ignorance.
29. What about cases in which an agent *has* ruled out the proposition Q? Should rational agents assign credences conditional on conditions that they've ruled out? For discussion and references on this question, see Titelbaum (2013a, Ch. 5).
30. I was careful to define the Ratio Formula so that it simply goes silent when $\text{cr}(Q) = 0$, and is therefore in need of *supplementation* if we want to constrain values like $\text{cr}(2 \mid 2)$. Other authors define the Ratio Formula so that it contains the same equation as ours but leaves off the restriction to $\text{cr}(Q) > 0$ cases. This forces an impossible calculation when $\text{cr}(Q) = 0$. Alternatively, one can leave the Ratio Formula unrestricted but make its equation $\text{cr}(P \mid Q) \cdot \text{cr}(Q) = \text{cr}(P \,\&\, Q)$. This has the advantage of being *true* even when $\text{cr}(Q) = 0$ (because $\text{cr}(P \,\&\, Q)$ will presumably equal 0 as well), but does no better than our Ratio Formula in constraining the value of $\text{cr}(2 \mid 2)$. (Any value we fill in for that conditional credence will make the relevant product-equation true.)
31. For a historical overview of the approach and detailed comparison of the disparate formal systems, see Makinson (2011).
32. Seidenfeld, Schervish, and Kadane (2017) shows that this pattern generalizes: At each infinite cardinality, we cannot secure the relevant Conglomerability principle with Additivity principles of lower cardinalities; Conglomerability at a particular level requires Additivity at that same level.
33. I got the example that follows from Brian Weatherson.
34. Contrast our move from comparative to quantitative representations of doxastic attitudes in Chapter 1. There the additional structure of a numerical representation allowed us to model features like confidence-gap sizes, which plausibly make a difference to agents' real-world decisions.
35. Let me quickly tie up one loose end: This section discussed cases in which it might be rational for an agent to assign unconditional credence 0 to a proposition without ruling it out. All the cases in which this might be rational involve credence assignments over infinite partitions. For the rest of this book we will be working with finite partitions, and will revert to the assumption we were making prior to this section that credence 0 always represents ruling something out.
36. Actually, Jeffrey's original proposal was a bit more complicated than that. In Jeffrey (1965) he began with a set of propositions $\{B_1, B_2, \ldots, B_n\}$ in which the credence change originated, but did not require the B_m to form a partition. Instead, he constructed a set of "atoms", which we can think of as state-descriptions constructed from the B_m. (Each atom was a consistent conjunction in which each B_m appeared exactly once, either affirmed or negated.) The Rigidity condition (which Jeffrey sometimes called "invariance") and Jeffrey Conditionalization were then applied to these atoms rather than directly to the B_m in which the credence change originated.

Notice that in this construction the atoms form a partition. Further, Jeffrey recognized that if the B_m themselves formed a partition, the atoms wound up in a one-to-one correspondence with the B_m to which they were logically equivalent. I think it's for this reason that Jeffrey later (2004, Ch. 3) dropped the business with "atoms" and applied his probability kinematics directly to any finite partition.

37. Though see Lange (2000) for an argument that this order-dependence is not a problem because the character of the experiences changes when they're temporally rearranged.
38. Interestingly, the main thrust of van Fraassen's article is that while Maximum Entropy *is* capable of providing a solution to the Judy Benjamin Problem, that solution is intuitively unappealing.
39. Because we're going to be using the entropy values only for comparative purposes, in the end it won't make a difference what base we use for the logarithms in Equation (5.13). But just to make your answers easily checkable with others', please follow Jaynes (1957a) in using the natural log ln.
40. I owe this problem to Sarah Moss.

Bibliography of Volumes 1 & 2

Achinstein, Peter (1963). Variety and Analogy in Confirmation Theory. *Philosophy of Science* 3, pp. 207–21.

Adams, Ernest (1962). On Rational Betting Systems. *Archiv für mathematische Logik und Grundlagenforschung* 6, pp. 7–29.

Adams, Ernest (1965). The Logic of Conditionals. *Inquiry* 8, pp. 166–97.

Alchourrón, Carlos E., Peter Gärdenfors, and David Makinson (1985). On the Logic of Theory Change: Partial Meet Contraction and Revision Functions. *The Journal of Symbolic Logic* 50, pp. 510–30.

Allais, Maurice (1953). Le comportement de l'homme rationnel devant le risque: Critique des postulates et axiomes de l'ecole Américaine. *Econometrica* 21, pp. 503–46.

Amrhein, Valentin, Sander Greenland, and Blake McShane (2019). Scientists Rise up against Statistical Significance. *Nature* 567, pp. 305–7.

Armendt, Brad (1980). Is There a Dutch Book Argument for Probability Kinematics? *Philosophy of Science* 47, pp. 583–8.

Armendt, Brad (1992). Dutch Strategies for Diachronic Rules: When Believers See the Sure Loss Coming. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 1, pp. 217–29.

Arntzenius, Frank (1993). The Common Cause Principle. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 2, pp. 227–37.

Arntzenius, Frank (2003). Some Problems for Conditionalization and Reflection. *The Journal of Philosophy* 100, pp. 356–70.

Arrow, Kenneth J. (1951). *Social Choice and Individual Values.* New York: John Wiley and Sons.

Bandyopadhyay, Prasanta S. and Malcolm R. Forster (2011). *Philosophy of Statistics.* Vol. 7. Handbook of the Philosophy of Science. Amsterdam: Elsevier.

Barnett, Jeffrey A. (1981). Computational Methods for a Mathematical Theory of Evidence. In: *Proceedings of the 7th International Joint Conference on AI,* pp. 868–75.

Bartha, Paul and Christopher R. Hitchcock (1999). No One Knows the Date or the Hour: An Unorthodox Application of Rev. Bayes's Theorem. *Philosophy of Science* 66, S339–53.

Berger, James O. and Donald A. Berry (1988). Statistical Analysis and the Illusion of Objectivity. *American Scientist* 76, pp. 159–65.

Bergmann, Merrie, James Moor, and Jack Nelson (2013). *The Logic Book.* 6th edition. New York: McGraw Hill.

Berker, Selim (2013). Epistemic Teleology and the Separateness of Propositions. *Philosophical Review* 122, pp. 337–93.

Bernoulli, Daniel (1738/1954). Exposition of a New Theory on the Measurement of Risk. *Econometrica* 22, pp. 23–36.

Bernoulli, Jacob (1713). *Ars Conjectandi.* Basiliae.

Bertrand, Joseph (1888/1972). *Calcul des probabilités.* 2nd edition. New York: Chelsea Publishing Company.

Bickel, P.J., E.A. Hammel, and J.W. O'Connell (1975). Sex Bias in Graduate Admissions: Data from Berkeley. *Science* 187, pp. 398–404.

Birnbaum, Allan (1962). On the Foundations of Statistical Inference. *Journal of the American Statistical Association* 57, pp. 269–306.

Bolzano, Bernard (1837/1973). *Wissenschaftslehre*. Translated by Jan Berg under the title *Theory of Science*. Dordrecht: Reidel.

Bovens, Luc and Stephan Hartmann (2003). *Bayesian Epistemology*. Oxford: Oxford University Press.

Bradley, Darren (2010). Conditionalization and Belief *De Se*. *Dialectica* 64, pp. 247–50.

Bradley, Darren (2011). Self-location Is No Problem for Conditionalization. *Synthese* 182, pp. 393–411.

Bradley, Darren (2015). *A Criticial Introduction to Formal Epistemology*. London: Bloomsbury.

Bratman, Michael E. (1987). *Intention, Plans, and Practical Reason*. Cambridge, MA: Harvard University Press.

Brier, George (1950). Verification of Forecasts Expressed in Terms of Probability. *Monthly Weather Review* 78, pp. 1–3.

Briggs, R.A. (2010). Putting a Value on Beauty. In: *Oxford Studies in Epistemology*. Ed. by Tamar Szabó Gendler and John Hawthorne. Vol. 3. Oxford University Press, pp. 3–34.

Briggs, R.A. and Richard Pettigrew (2020). An Accuracy-Dominance Argument for Conditionalization. *Noûs* 54, pp. 162–81.

Broome, John (1999). Normative Requirements. *Ratio* 12, pp. 398–419.

Buchak, Lara (2013). *Risk and Rationality*. Oxford: Oxford University Press.

Campbell-Moore, Catrin and Benjamin A. Levinstein (2021). Strict Propriety Is Weak. *Analysis* 81.1, pp. 8–13.

Capotorti, Andrea and Barbara Vantaggi (2000). Axiomatic Characterization of Partial Ordinal Relations. *International Journal of Approximate Reasoning* 24, pp. 207–19.

Carnap, Rudolf (1945). On Inductive Logic. *Philosophy of Science* 12, pp. 72–97.

Carnap, Rudolf (1947). On the Application of Inductive Logic. *Philosophy and Phenomenological Research* 8, pp. 133–48.

Carnap, Rudolf (1950). *Logical Foundations of Probability*. Chicago: University of Chicago Press.

Carnap, Rudolf (1955/1989). Statistical and Inductive Probability. In: *Readings in the Philosophy of Science*. Ed. by Baruch A. Brody and Richard E. Grandy. 2nd edition. Hoboken: Prentice-Hall, pp. 279–87.

Carnap, Rudolf (1962a). *Logical Foundations of Probability*. 2nd edition. Chicago: University of Chicago Press.

Carnap, Rudolf (1962b). The Aim of Inductive Logic. In: *Logic, Methodology, and the Philosophy of Science*. Ed. by P. Suppes, E. Nagel, and A. Tarski. Stanford University: Stanford University Press, pp. 303–18.

Carnap, Rudolf (1971). A Basic System of Inductive Logic, Part 1. In: *Studies in Inductive Logic and Probability*. Ed. by Rudolf Carnap and Richard C. Jeffrey. Vol. I. Berkeley: University of California Press, pp. 33–166.

Carnap, Rudolf (1980). A Basic System of Inductive Logic, Part 2. In: *Studies in Inductive Logic and Probability*. Ed. by Richard C. Jeffrey. Vol. II. Berkeley: University of California Press, pp. 7–156.

Carr, Jennifer (2017). Epistemic Utility Theory and the Aim of Belief. *Philosophy and Phenomenological Research* 95, pp. 511–34.

Cartwright, Nancy (1979). Causal Laws and Effective Strategies. *Noûs* 13, pp. 419–37.
Carver, Ronald P. (1978). The Case against Statistical Significance Testing. *Harvard Educational Review* 48, pp. 378–99.
Chandler, Jake (2013). Contrastive Confirmation: Some Competing Accounts. *Synthese* 190, pp. 129–38.
Cherniak, Christopher (1986). *Minimal Rationality.* Cambridge, MA: The MIT Press.
Chihara, C. (1981). Quine and the Confirmational Paradoxes. In: *Midwest Studies in Philosophy 6: Foundations of Analytic Philosophy.* Ed. by P. French, H. Wettstein, and T. Uehling. Minneapolis: University of Minnesota Press, pp. 425–52.
Christensen, David (1991). Clever Bookies and Coherent Beliefs. *The Philosophical Review* 100, pp. 229–47.
Christensen, David (1999). Measuring Confirmation. *The Journal of Philosophy* 96, pp. 437–61.
Christensen, David (2001). Preference-Based Arguments for Probabilism. *Philosophy of Science* 68, pp. 356–76.
Christensen, David (2004). *Putting Logic in its Place.* Oxford: Oxford University Press.
Colyvan, Mark (2004). The Philosophical Significance of Cox's Theorem. *International Journal of Approximate Reasoning* 37, pp. 71–85.
Cox, Richard T. (1946). Probability, Frequency and Reasonable Expectation. *American Journal of Physics* 14, pp. 1–13.
Cox, Richard T. (1961). *The Algebra of Probable Inference.* Baltimore, MD: The Johns Hopkins Press.
Crupi, Vincenzo, Branden Fitelson, and Katya Tentori (2008). Probability, Confirmation, and the Conjunction Fallacy. *Thinking & Reasoning* 14, pp. 182–99.
Crupi, Vincenzo, Katya Tentori, and Michel Gonzalez (2007). On Bayesian Measures of Evidential Support: Theoretical and Empirical Issues. *Philosophy of Science* 74, pp. 229–52.
Davidson, Donald (1984). *Inquiries into Truth and Interpretation.* Oxford: Clarendon Press.
Davidson, Donald, J.C.C. McKinsey, and Patrick Suppes (1955). Outlines of a Formal Theory of Value, I. *Philosophy of Science* 22, pp. 14–60.
de Finetti, Bruno (1931/1989). Probabilism: A Critical Essay on the Theory of Probability and the Value of Science. *Erkenntnis* 31, pp. 169–223. Translation of B. de Finetti, *Probabilismo,* Logos 14, pp. 163–219.
de Finetti, Bruno (1937/1964). Foresight: Its Logical Laws, its Subjective Sources. In: *Studies in Subjective Probability.* Ed. by Henry E. Kyburg Jr and H.E. Smokler. New York: Wiley, pp. 94–158. Originally published as "La prévision; ses lois logiques, ses sources subjectives" in *Annales de l'Institut Henri Poincaré* 7, pp. 1–68.
de Finetti, Bruno (1949/1951). La 'Logica del Plausible' Secondo la Conçezione di Polya. *Atti della XLII Riunione, Societa Italiana per il Progresso delle Scienze.* Presented in 1949, published in 1951, pp. 227–36.
de Finetti, Bruno (1974). *Theory of Probability.* Vol. 1. New York: Wiley.
de Finetti, Bruno (1995). *Filosofia della probabilità.* Ed. by Alberto Mura. Milan: Il Saggiatore.
DeGroot, Morris H. (1970). *Optimal Statistical Decisions.* Hoboken, New Jersey: Wiley.
Dempster, Arthur P. (1966). New Methods for Reasoning towards Posterior Distributions Based on Sample Data. *Annals of Mathematical Statistics* 37, pp. 355–74.
Dickson, Michael and Davis Baird (2011). Significance Testing. In: *Philosophy of Statistics.* Ed. by Prasanta S. Bandyopadhyay and Malcolm R. Forster. Vol. 7. Handbook of the Philosophy of Science. Amsterdam: Elsevier.

Domotor, Zoltan (1969). *Probabilistic Relational Structures and their Applications.* Technical Report 144. Stanford University: Institute for Mathematical Studies in the Social Sciences.

Earman, John (1992). *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory.* Cambridge, MA: The MIT Press.

Easwaran, Kenny (2013). Expected Accuracy Supports Conditionalization—and Conglomerability and Reflection. *Philosophy of Science* 80, pp. 119–42.

Easwaran, Kenny (2014a). Decision Theory without Representation Theorems. *Philosophers' Imprint* 14, pp. 1–30.

Easwaran, Kenny (2014b). Regularity and Hyperreal Credences. *Philosophical Review* 123, pp. 1–41.

Eddington, A. (1939). *The Philosophy of Physical Science.* Cambridge: Cambridge University Press.

Edwards, A.W.F. (1972). *Likelihood: An Account of the Statistical Concept of Likelihood and its Application to Scientific Inference.* Cambridge: Cambridge University Press.

Eells, Ellery (1982). *Rational Decision and Causality.* Cambridge Studies in Philosophy. Cambridge: Cambridge University Press.

Eells, Ellery (1985). Problems of Old Evidence. *Pacific Philosophical Quarterly* 66, pp. 283–302.

Eells, Ellery and Branden Fitelson (2000). Measuring Confirmation and Evidence. *Journal of Philosophy* 97, pp. 663–72.

Eells, Ellery and Branden Fitelson (2002). Symmetries and Asymmetries in Evidential Support. *Philosophical Studies* 107, pp. 129–42.

Efron, B. (1986). Why Isn't Everyone a Bayesian? *The American Statistician* 40, pp. 1–5.

Egan, Andy (2007). Some Counterexamples to Causal Decision Theory. *Philosophical Review* 116, pp. 93–114.

Egan, Andy and Michael G. Titelbaum (2022). Self-locating Belief. Forthcoming in *The Stanford Encyclopedia of Philosophy.*

Elga, Adam (2000). Self-locating Belief and the Sleeping Beauty Problem. *Analysis* 60, pp. 143–7.

Elga, Adam (2007). Reflection and Disagreement. *Noûs* 41, pp. 478–502.

Elga, Adam (2010). Subjective Probabilities Should Be Sharp. *Philosophers' Imprint* 10, pp. 1–11.

Ellenberg, Jordan (2014). *How Not to Be Wrong: The Power of Mathematical Thinking.* New York: Penguin Press.

Ellis, Robert Leslie (1849). On the Foundations of the Theory of Probabilities. *Transactions of the Cambridge Philosophical Society* VIII, pp. 1–6.

Ellsberg, Daniel (1961). Risk, Ambiguity, and the Savage Axioms. *The Quarterly Journal of Economics* 75, pp. 643–69.

Feldman, Richard (2007). Reasonable Religious Disagreements. In: *Philosophers without Gods: Meditations on Atheism and the Secular Life.* Ed. by Louise M. Antony. Oxford: Oxford University Press, pp. 194–214.

Feller, William (1968). *An Introduction to Probability Theory and its Applications.* 3rd edition. New York: Wiley.

Fermat, Pierre and Blaise Pascal (1654/1929). Fermat and Pascal on Probability. In: *A Source Book in Mathematics.* Ed. by D. Smith. Translated by Vera Sanford. New York: McGraw-Hill, pp. 546–65.

Fine, Terrence L. (1973). *Theories of Probability: An Examination of Foundations.* New York, London: Academic Press.

Fishburn, Peter C. (1981). Subjective Expected Utility: A Review of Normative Theories. *Theory and Decision* 13, pp. 129–99.

Fishburn, Peter C. (1986). The Axioms of Subjective Probability. *Statistical Science* 1, pp. 335–45.

Fisher, Ronald A. (1956). *Statistical Methods and Scientific Inference.* Edinburgh: Oliver and Boyd.

Fitelson, Branden (2006). Logical Foundations of Evidential Support. *Philosophy of Science* 73, pp. 500–12.

Fitelson, Branden (2007). Likelihoodism, Bayesianism, and Relational Confirmation. *Synthese* 156, pp. 473–89.

Fitelson, Branden (2008). A Decision Procedure for Probability Calculus with Applications. *The Review of Symbolic Logic* 1, pp. 111–25.

Fitelson, Branden (2012). Evidence of Evidence Is Not (Necessarily) Evidence. *Analysis* 72, pp. 85–8.

Fitelson, Branden (2015). The Strongest Possible Lewisian Triviality Result. *Thought* 4, pp. 69–74.

Fitelson, Branden and Alan Hájek (2014). Declarations of Independence. *Synthese* 194, pp. 3979–95.

Fitelson, Branden and James Hawthorne (2010a). How Bayesian Confirmation Theory Handles the Paradox of the Ravens. *The Place of Probability in Science*. Ed. by Ellery Eells and J. Fetzer. *Boston Studies in the Philosophy of Science* 284, pp. 247–75.

Fitelson, Branden and James Hawthorne (2010b). The Wason Task(s) and the Paradox of Confirmation. *Philosophical Perspectives* 24, pp. 207–41.

Fitelson, Branden and David McCarthy (ms). Accuracy and Comparative Likelihood. Unpublished manuscript.

Fitelson, Branden and Daniel Osherson (2015). Remarks on "Random Sequences". *Australasian Journal of Logic* 12, pp. 11–16.

Foley, Richard (1987). *The Theory of Epistemic Rationality.* Cambridge, MA: Harvard University Press.

Foley, Richard (1993). *Working without a Net.* Oxford: Oxford University Press.

Foley, Richard (2009). Beliefs, Degrees of Belief, and the Lockean Thesis. In: *Degrees of Belief.* Ed. by Franz Huber and Christoph Schmidt-Petri. Vol. 342. Synthese Library. Springer, pp. 37–48.

Frigerio, Roberta et al. (2005). Education and Occupations Preceding Parkinson's Disease. *Neurology* 65, pp. 1575–83.

Gaifman, H. and M. Snir (1982). Probabilities over Rich Languages. *Journal of Symbolic Logic* 47, pp. 495–548.

Gaifman, Haim (2004). Reasoning with Limited Resources and Assigning Probabilities to Arithmetical Statements. *Synthese* 140, pp. 97–119.

Galavotti, Maria Carla (2005). *Philosophical Introduction to Probability.* CSLI Lecture Notes 167. Stanford, CA: CSLI Publications.

Gallo, Valentina et al. (2018). Exploring Causality of the Association between Smoking and Parkinson's Disease. *International Journal of Epidemiology* 48, pp. 912–25.

Gandenberger, Greg (2014). *Titelbaum's Counterexample to the Law of Likelihood.* Blog post available at URL: https://gandenberger.org/2014/05/26/titelbaum-counterexample/.

Gandenberger, Greg (2015). A New Proof of the Likelihood Principle. *British Journal for the Philosophy of Science* 66, pp. 475–503.

Gandenberger, Greg (ms). New Responses to Three Purported Counterexamples to the Likelihood Principle. Unpublished manuscript.

Garber, Daniel (1983). Old Evidence and Logical Omniscience in Bayesian Confirmation Theory. In: *Testing Scientific Theories*. Ed. by John Earman. Vol. 10. Minnesota Studies in the Philosophy of Science. Minneapolis: University of Minnesota Press, pp. 99–132.

Gibbard, A. and W. Harper (1978/1981). Counterfactuals and Two Kinds of Expected Utility. In: *Ifs: Conditionals, Belief, Decision, Chance, and Time*. Ed. by W. Harper, Robert C. Stalnaker, and G. Pearce. Dordrecht: Reidel, pp. 153–90.

Gillies, Donald (2000). Varieties of Propensity. *British Journal for the Philosophy of Science* 51, pp. 807–35.

Glass, David H. and Mark McCartney (2015). A New Argument for the Likelihood Ratio Measure of Confirmation. *Acta Analytica* 30, pp. 59–65.

Glymour, Clark (1980). *Theory and Evidence*. Princeton, NJ: Princeton University Press.

Good, I.J. (1952). Rational Decisions. *Journal of the Royal Statistical Society, Series B* 14, pp. 107–14.

Good, I.J. (1962). Subjective Probability as the Measure of a Non-measurable Set. In: *Logic, Methodology, and the Philosophy of Science*. Ed. by P. Suppes, E. Nagel, and A. Tarski. Stanford, CA: Stanford University Press, pp. 319–29.

Good, I.J. (1967). The White Shoe Is a Red Herring. *British Journal for the Philosophy of Science* 17, p. 322.

Good, I.J. (1968). The White Shoe qua Herring Is Pink. *British Journal for the Philosophy of Science* 19, pp. 156–7.

Good, I.J. (1971). Letter to the Editor. *The American Statistician* 25, pp. 62–3.

Goodman, Nelson (1946). A Query on Confirmation. *The Journal of Philosophy* 43, pp. 383–5.

Goodman, Nelson (1955). *Fact, Fiction, and Forecast*. Cambridge, MA: Harvard University Press.

Greaves, Hilary (2013). Epistemic Decision Theory. *Mind* 122, pp. 915–52.

Greaves, Hilary and David Wallace (2006). Justifying Conditionalization: Conditionalization Maximizes Expected Epistemic Utility. *Mind* 115, pp. 607–32.

Hacking, Ian (1965). *The Logic of Statistical Inference*. Cambridge: Cambridge University Press.

Hacking, Ian (1967). Slightly More Realistic Personal Probability. *Philosophy of Science* 34, pp. 311–25.

Hacking, Ian (1971). The Leibniz-Carnap Program for Inductive Logic. *The Journal of Philosophy* 68, pp. 597–610.

Hacking, Ian (2001). *An Introduction to Probability and Inductive Logic*. Cambridge: Cambridge University Press.

Hájek, Alan (1996). 'Mises Redux'—Redux: Fifteen Arguments against Finite Frequentism. *Erkenntnis* 45, pp. 209–27.

Hájek, Alan (2003). What Conditional Probability Could Not Be. *Synthese* 137, pp. 273–323.

Hájek, Alan (2009a). Arguments for—or against—Probabilism? In: *Degrees of Belief*. Ed. by Franz Huber and Christoph Schmidt-Petri. Vol. 342. Synthese Library. Springer, pp. 229–51.

Hájek, Alan (2009b). Fifteen Arguments against Hypothetical Frequentism. *Erkenntnis* 70, pp. 211–35.

Hájek, Alan (2011a). Conditional Probability. In: *Philosophy of Statistics*. Ed. by Prasanta S. Bandyopadhyay and Malcolm R. Forster. Vol. 7. Handbook of the Philosophy of Science. Amsterdam: Elsevier, pp. 99–136.

Hájek, Alan (2011b). Triviality Pursuit. *Topoi* 30, pp. 3–15.

Hájek, Alan (2019). Interpretations of Probability. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Fall 2019. URL: http://plato.stanford.edu/archives/fall2019/entries/probability-interpret/.

Hájek, Alan and James M. Joyce (2008). Confirmation. In: *The Routledge Companion to Philosophy of Science.* Ed. by Stathis Psillos and Martin Curd. New York: Routledge, pp. 115–28.

Hall, Ned (2004). Two Mistakes about Credence and Chance. *Australasian Journal of Philosophy* 82, pp. 93–111.

Halpern, Joseph Y. (1999). Cox's Theorem Revisited. *Journal of Artificial Intelligence Research* 11, pp. 429–35.

Halpern, Joseph Y. (2003). *Reasoning about Uncertainty.* Cambridge, MA: MIT Press.

Halpern, Joseph Y. (2004). Sleeping Beauty Reconsidered: Conditioning and Reflection in Asynchronous Systems. *Proceedings of the Twentieth Conference on Uncertainty in AI,* pp. 226–34.

Halpern, Joseph Y. (2005). Sleeping Beauty Reconsidered: Conditioning and Reflection in Asynchronous Systems. In: *Oxford Studies in Epistemology.* Ed. by Tamar Szabó Gendler and John Hawthorne. Vol. 1. Oxford: Oxford University Press, pp. 111–42.

Harman, Gilbert (1986). *Change in View.* Boston: The MIT Press.

Harrison-Trainor, Matthew, Wesley H. Holliday, and Thomas F. Icard III (2016). A Note on Cancellation Axioms for Comparative Probability. *Theory and Decision* 80, pp. 159–66.

Hart, Casey and Michael G. Titelbaum (2015). Intuitive Dilation? *Thought* 4, pp. 252–62.

Haverkamp, Nick and Moritz Schulz (2012). A Note on Comparative Probability. *Erkenntnis* 76, pp. 395–402.

Hawthorne, James (2014). Inductive Logic. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Winter 2014. URL: http://plato.stanford.edu/archives/win2014/entries/logic-inductive/.

Hawthorne, James (2016). A Logic of Comparative Support: Qualitative Conditional Probability Relations Representable by Popper Functions. In: *The Oxford Handbook of Probability and Philosophy.* Ed. by Alan Hájek and Christopher R. Hitchcock. Oxford: Oxford University Press, pp. 277–95.

Hawthorne, James and Branden Fitelson (2004). Re-solving Irrelevant Conjunction with Probabilistic Independence. *Philosophy of Science* 71, pp. 505–14.

Hedden, Brian (2013). Incoherence without Exploitability. *Noûs* 47, pp. 482–95.

Hempel, Carl G. (1945a). Studies in the Logic of Confirmation (I). *Mind* 54, pp. 1–26.

Hempel, Carl G. (1945b). Studies in the Logic of Confirmation (II). *Mind* 54, pp. 97–121.

Hesse, Mary (1963). *Models and Analogies in Science.* London: Sheed & Ward.

Heukelom, Floris (2015). A History of the Allais Paradox. *The British Journal for the History of Science* 48, pp. 147–69.

Hintikka, Jaakko (1975). Impossible Possible Worlds Vindicated. *Journal of Philosophical Logic* 4, pp. 475–84.

Hitchcock, Christopher R. (2004). Beauty and the Bets. *Synthese* 139, pp. 405–20.

Hitchcock, Christopher R. (2021). Probabilistic Causation. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Spring 2021. URL: https://plato.stanford.edu/archives/spr2021/entries/causation-probabilistic/.

Holton, Richard (2014). Intention as a Model for Belief. In: *Rational and Social Agency: The Philosophy of Michael Bratman.* Ed. by Manuel Vargas and Gideon Yaffe. Oxford: Oxford University Press, pp. 12–37.

Hooker, C.A. (1968). Goodman, 'Grue' and Hempel. *Philosophy of Science* 35, pp. 232–47.

Hosiasson-Lindenbaum, Janina (1940). On Confirmation. *Journal of Symbolic Logic* 5, pp. 133–48.

Howson, Colin (1992). Dutch Book Arguments and Consistency. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 2, pp. 161–8.

Howson, Colin (2014). Finite Additivity, Another Lottery Paradox and Conditionalisation. *Synthese* 191, pp. 989–1012.

Howson, Colin and Peter Urbach (2006). *Scientific Reasoning: The Bayesian Approach.* 3rd edition. Chicago: Open Court.

Huber, Franz (2016). Formal Representations of Belief. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Spring 2016. URL: https://plato.stanford.edu/archives/spr2016/entries/formal-belief/.

Hume, David (1739–40/1978). *A Treatise of Human Nature.* Ed. by L.A. Selby-Bigge and Peter H. Nidditch. 2nd edition. Oxford: Oxford University Press.

Humphreys, Paul (1985). Why Propensities Cannot Be Probabilities. *Philosophical Review* 94, pp. 557–70.

Icard III, Thomas F. (2016). Pragmatic Considerations on Comparative Probability. *Philosophy of Science* 83, pp. 348–70.

Jackson, Elizabeth G. (2020). The Relationship between Belief and Credence. *Philosophy Compass* 15, pp. 1–13.

Jaynes, E.T. (1957a). Information Theory and Statistical Mechanics I. *Physical Review* 106, pp. 62–30.

Jaynes, E.T. (1957b). Information Theory and Statistical Mechanics II. *Physical Review* 108, pp. 171–90.

Jaynes, E.T. (1968). Prior Probabilities. *IEEE Transactions on Systems Science and Cybernetics* SEC-4, pp. 227–41.

Jeffrey, Richard C. (1965). *The Logic of Decision.* 1st edition. McGraw-Hill Series in Probability and Statistics. New York: McGraw-Hill.

Jeffrey, Richard C. (1970). Dracula Meets Wolfman: Acceptance vs. Partial Belief. In: *Induction, Acceptance, and Rational Belief.* Ed. by M. Swain. Dordrecht: Reidel, pp. 157–85.

Jeffrey, Richard C. (1983). *The Logic of Decision.* 2nd edition. Chicago: University of Chicago Press.

Jeffrey, Richard C. (1987). Indefinite Probability Judgment: A Reply to Levi. *Philosophy of Science* 54, pp. 586–91.

Jeffrey, Richard C. (1993). Causality and the Logic of Decision. *Philosophical Topics* 21, pp. 139–51.

Jeffrey, Richard C. (2004). *Subjective Probability: The Real Thing.* Cambridge: Cambridge University Press.

Johnson, W.E. (1932). Probability: The Deductive and Inductive Problems. *Mind* 41, pp. 409–23.

Joyce, James M. (1998). A Nonpragmatic Vindication of Probabilism. *Philosophy of Science* 65, pp. 575–603.

Joyce, James M. (1999). *The Foundations of Causal Decision Theory.* Cambridge: Cambridge University Press.

Joyce, James M. (2005). How Probabilities Reflect Evidence. *Philosophical Perspectives* 19, pp. 153–78.

Joyce, James M. (2009). Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial Belief. In: *Degrees of Belief.* Ed. by Franz Huber and Christoph Schmidt-Petri. Vol. 342. Synthese Library. Springer, pp. 263–97.

Joyce, James M. (2010). A Defense of Imprecise Credences in Inference and Decision Making. *Philosophical Perspectives* 24, pp. 281–323.

Kahneman, Daniel and Amos Tversky (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica* XLVII, pp. 263–91.

Kaplan, Mark (1996). *Decision Theory as Philosophy.* Cambridge: Cambridge University Press.

Kelley, Mikayla (ms). On Accuracy and Coherence with Infinite Opinion Sets. Unpublished manuscript.

Kemeny, John G. (1955). Fair Bets and Inductive Probabilities. *The Journal of Symbolic Logic* 20, pp. 263–73.

Kemeny, John G. and Paul Oppenheim (1952). Degree of Factual Support. *Philosophy of Science* 19, pp. 307–24.

Keynes, John Maynard (1921). *Treatise on Probability.* London: Macmillan and Co., Limited.

Keynes, John Maynard (1923). *A Tract on Monetary Reform.* Macmillan and Co., Limited.

Keynes, John Maynard (1937). The General Theory of Employment. *The Quarterly Journal of Economics* 51, pp. 209–23.

Kierland, Brian and Bradley Monton (2005). Minimizing Inaccuracy for Self-locating Beliefs. *Philosophy and Phenomenological Research* 70, pp. 384–95.

Kim, Jaegwon (1988). What Is "Naturalized Epistemology"? *Philosophical Perspectives* 2, pp. 381–405.

Kim, Namjoong (2009). Sleeping Beauty and Shifted Jeffrey Conditionalization. *Synthese* 168, pp. 295–312.

Knight, Frank Hyneman (1921). *Risk, Uncertainty and Profit.* Boston and New York: Houghton Mifflin.

Kolmogorov, A.N. (1933/1950). *Foundations of the Theory of Probability.* Translation edited by Nathan Morrison. New York: Chelsea Publishing Company.

Konek, Jason and Benjamin A. Levinstein (2019). The Foundations of Epistemic Decision Theory. *Mind* 128, pp. 69–107.

Kornblith, Hilary (1993). Epistemic Normativity. *Synthese* 94, pp. 357–76.

Kraft, Charles H., John W. Pratt, and A. Seidenberg (1959). Intuitive Probability on Finite Sets. *The Annals of Mathematical Statistics* 30, pp. 408–19.

Krantz, D.H. et al. (1971). *Foundations of Measurement, Vol 1: Additive and Polynomial Representations.* Cambridge: Academic Press.

Kuhn, Thomas S. (1957). *The Copernican Revolution: Planetary Astronomy in the Development of Western Thought.* New York: MJF Books.

Kyburg Jr, Henry E. (1961). *Probability and the Logic of Rational Belief.* Middletown: Wesleyan University Press.

Kyburg Jr, Henry E. (1970). Conjunctivitis. In: *Induction, Acceptance, and Rational Belief.* Ed. by M. Swain. Boston: Reidel, pp. 55–82.

Laddaga, R. (1977). Lehrer and the Consensus Proposal. *Synthese* 36, pp. 473–7.

Lance, Mark Norris (1995). Subjective Probability and Acceptance. *Philosophical Studies* 77, pp. 147–79.

Lange, Alexandra (2019). Can Data Be Human? The Work of Giorgia Lupi. *The New Yorker.* Published May 25, 2019.

Lange, Marc (2000). Is Jeffrey Conditionalization Defective by Virtue of Being Non-commutative? Remarks on the Sameness of Sensory Experience. *Synthese* 123, pp. 393–403.

Laplace, Pierre-Simon (1814/1995). *Philosophical Essay on Probabilities.* Translated from the French by Andrew Dale. New York: Springer.

Lehman, R. Sherman (1955). On Confirmation and Rational Betting. *Journal of Symbolic Logic* 20, pp. 251–62.

Lehrer, K. and Carl Wagner (1983). Probability Amalgamation and the Independence Issue: A Reply to Laddaga. *Synthese* 55, pp. 339–46.

Leitgeb, Hannes and Richard Pettigrew (2010a). An Objective Justification of Bayesianism I: Measuring Inaccuracy. *Philosophy of Science* 77, pp. 201–35.

Leitgeb, Hannes and Richard Pettigrew (2010b). An Objective Justification of Bayesianism II: The Consequences of Minimizing Inaccuracy. *Philosophy of Science* 77, pp. 236–72.

Lele, Subhash R. (2004). Evidence Functions and the Optimality of the Law of Likelihood. In: *The Nature of Scientific Evidence: Statistical, Philosophical, and Empirical Considerations*. Chicago: University of Chicago Press, pp. 191–216.

Levi, Isaac (1974). On Indeterminate Probabilities. *The Journal of Philosophy* 71, pp. 391–418.

Levi, Isaac (1980). *The Enterprise of Knowledge*. Boston: The MIT Press.

Levi, Isaac (1987). The Demons of Decision. *The Monist* 70, pp. 193–211.

Lewis, C.I. (1946). *An Analysis of Knowledge and Valuation*. La Salle, IL: Open Court.

Lewis, David (1971). Immodest Inductive Methods. *Philosophy of Science* 38, pp. 54–63.

Lewis, David (1976). Probabilities of Conditionals and Conditional Probabilities. *The Philosophical Review* 85, pp. 297–315.

Lewis, David (1979). Atittudes *de dicto* and *de se*. *The Philosophical Review* 88, pp. 513–43.

Lewis, David (1980). A Subjectivist's Guide to Objective Chance. In: *Studies in Inductive Logic and Probability*. Ed. by Richard C. Jeffrey. Vol. 2. Berkeley: University of California Press, pp. 263–94.

Lewis, David (1981a). Causal Decision Theory. *Australasian Journal of Philosophy* 59, pp. 5–30.

Lewis, David (1981b). 'Why Ain'cha Rich?' *Noûs* 15, pp. 377–80.

Lewis, David (1994). Humean Supervenience Debugged. *Mind* 103, pp. 473–90.

Lewis, David (1996). Desire as Belief II. *Mind* 105, pp. 303–13.

Lewis, David (2001). Sleeping Beauty: Reply to Elga. *Analysis* 61, pp. 171–6.

Lichtenstein, S., B. Fischoff, and L. Phillips (1982). Calibration of Probabilities: The State of the Art to 1980. In: *Judgment under Uncertainty: Heuristics and Biases*. Ed. by Daniel Kahneman, P. Slovic, and Amos Tversky. Cambridge: Cambridge University Press, pp. 306–34.

Lindley, Dennis V. (1982). Scoring Rules and the Inevitability of Probability. *International Statistical Review* 50, pp. 1–26.

Lindley, Dennis V. (1985). *Making Decisions*. 2nd edition. London: Wiley.

Liu, Liping and Ronald R. Yager (2008). Classic Works of the Dempster-Shafer Theory of Belief Functions: An Introduction. In: *Classic Works of the Dempster-Shafer Theory of Belief Functions*. Ed. by Ronald R. Yager and Liping Liu. Vol. 219. Studies in Fuzziness and Soft Computing. Berlin: Springer, pp. 1–34.

Locke, John (1689/1975). *An Essay Concerning Human Understanding*. Ed. by Peter H. Nidditch. Oxford: Oxford University Press.

MacFarlane, John (2005). Making Sense of Relative Truth. *Proceedings of the Aristotelian Society* 105, pp. 321–39.

Maher, Patrick (1993). *Betting on Theories*. Cambridge Studies in Probability, Induction, and Decision Theory. Cambridge: Cambridge University Press.

Maher, Patrick (1996). Subjective and Objective Confirmation. *Philosophy of Science* 63, pp. 149–74.

Maher, Patrick (2002). Joyce's Argument for Probabilism. *Philosophy of Science* 96, pp. 73–81.

Maher, Patrick (2010). Explication of Inductive Probability. *Journal of Philosophical Logic* 39, pp. 593–616.

Makinson, David C. (1965). The Paradox of the Preface. *Analysis* 25, pp. 205–7.

Makinson, David C. (2011). Conditional Probability in the Light of Qualitative Belief Change. *Journal of Philosophical Logic* 40, pp. 121–53.

Mayo, Deborah (2018). *Statistical Inference as Severe Testing: How to Get beyond the Statistics Wars*. Cambridge: Cambridge University Press.

Mazurkiewicz, Stefan (1932). Zur Axiomatik der Wahrscheinlichkeitsrechnung. *Comptes rendues des séances de la Société des Sciences et des Lettres de Varsovie* 25, pp. 1–4.

Meacham, Christopher J. G. (2008). Sleeping Beauty and the Dynamics of De Se Beliefs. *Philosophical Studies* 138, pp. 245–70.

Meacham, Christopher J. G. (2010a). Unravelling the Tangled Web: Continuity, Internalism, Uniqueness, and Self-locating Belief. In: *Oxford Studies in Epistemology*. Ed. by Tamar Szabó Gendler and John Hawthorne. Vol. 3. Oxford: Oxford University Press, pp. 86–125.

Meacham, Christopher J.G. (2010b). Two Mistakes Regarding the Principal Principle. *British Journal for the Philosophy of Science* 61, pp. 407–31.

Meacham, Christopher J. G. (2016). Ur-Priors, Conditionalization, and Ur-Prior Conditionalization. *Ergo* 3, pp. 444–92.

Meacham, Christopher J.G. and Jonathan Weisberg (2011). Representation Theorems and the Foundations of Decision Theory. *Australasian Journal of Philosophy* 89, pp. 641–63.

Mellor, D.H. (2013). Review of *Probability in the Philosophy of Religion*. *Analysis* 73, pp. 548–54.

Moore, David S., George P. McCabe, and Bruce A. Craig (2009). *Introduction to the Practice of Statistics*. 6th edition. New York: W.H. Freeman and Company.

Moore, G.E. (1939). Proof of an External World. *Proceedings of the British Academy* 25, pp. 273–300.

Moss, Sarah (2012). Updating as Communication. *Philosophy and Phenomenological Research* 85, pp. 225–48.

Moss, Sarah (2015). Credal Dilemmas. *Noûs* 49, pp. 665–83.

Moss, Sarah (2018). *Probabilistic Knowledge*. Oxford: Oxford University Press.

Murphy, Allan H. (1973). A New Vector Partition of the Probability Score. *Journal of Applied Meteorology* 12, pp. 595–600.

Murphy, Allan H. and Robert L. Winkler (1977). Reliability of Subjective Probability Forecasts of Precipitation and Temperature. *Journal of the Royal Statistical Society, Series C* 26, pp. 41–7.

Neal, Radford M. (2006). *Puzzles of Anthropic Reasoning Resolved Using Full Non-indexical Conditioning*. Tech. rep. 0607. Department of Statistics, University of Toronto.

Neyman, J. and Egon Pearson (1967). *Joint Statistical Papers*. Cambridge: Cambridge University Press.

Nicod, Jean (1930). *Foundations of Geometry and Induction*. Translated by Philip Wiener. New York: Harcourt, Brace and Company.

Nozick, Robert (1969). Newcomb's Problem and Two Principles of Choice. In: *Essays in Honor of Carl G. Hempel*. Synthese Library. Dordrecht: Reidel, pp. 114–15.

Open Science Collaboration (2015). Estimating the reproducibility of psychological science. *Science* 349. DOI: 10.1126/science.aac4716.

Papineau, David (2012). *Philosophical Devices: Proofs, Probabilities, Possibilities, and Sets*. Oxford: Oxford University Press.

Paris, J.B. (1994). *The Uncertain Reasoner's Companion*. Cambridge: Cambridge University Press.

Pascal, Blaise (1670/1910). *Pensées*. Translated by W.F. Trotter. London: Dent.

Pearson, K., A. Lee, and L. Bramley-Moore (1899). Genetic (Reproductive) Selection: Inheritance of Fertility in Man. *Philosophical Transactions of the Royal Society A* 73, pp. 534–39.

Pedersen, Arthur Paul and Gregory Wheeler (2014). Demystifying Dilation. *Erkenntnis* 79, pp. 1305–42.

Peirce, Charles Sanders (1910/1932). Notes on the Doctrine of Chances. In: *Collected Papers of Charles Sanders Peirce*. Ed. by Charles Hartshorne and Paul Weiss. Cambridge, MA: Harvard University Press, pp. 404–14.

Peterson, Martin (2009). *An Introduction to Decision Theory*. Cambridge Introductions to Philosophy. Cambridge: Cambridge University Press.

Pettigrew, Richard (2013a). A New Epistemic Utility Argument for the Principal Principle. *Episteme* 10, pp. 19–35.

Pettigrew, Richard (2013b). Epistemic Utility and Norms for Credences. *Philosophy Compass* 8, pp. 897–908.

Pettigrew, Richard (2014). Accuracy, Risk, and the Principle of Indifference. *Philosophy and Phenomenological Research* 92(1), pp. 35–59.

Pettigrew, Richard (2016). *Accuracy and the Laws of Credence*. Oxford: Oxford University Press.

Pettigrew, Richard (2021). On the Expected Utility Objection to the Dutch Book Argument for Probabilism. *Noûs* 55, pp. 23–38.

Pettigrew, Richard and Michael G. Titelbaum (2014). Deference Done Right. *Philosophers' Imprint* 14, pp. 1–19.

Pettigrew, Richard and Jonathan Weisberg, eds (2019). *The Open Handbook of Formal Epistemology*. Published open access online by PhilPapers. URL: https://philpapers.org/rec/PETTOH-2.

Piccione, Michele and Ariel Rubinstein (1997). On the Interpretation of Decision Problems with Imperfect Recall. *Games and Economic Behavior* 20, pp. 3–24.

Pollock, John L. (2001). Defeasible Reasoning with Variable Degrees of Justification. *Artificial Intelligence* 133, pp. 233–82.

Popper, Karl R. (1935/1959). *The Logic of Scientific Discovery*. London: Hutchinson & Co.

Popper, Karl R. (1938). A Set of Independent Axioms for Probability. *Mind* 47, pp. 275–9.

Popper, Karl R. (1954). Degree of Confirmation. *British Journal for the Philosophy of Science* 5, pp. 143–9.

Popper, Karl R. (1955). Two Autonomous Axiom Systems for the Calculus of Probabilities. *British Journal for the Philosophy of Science* 6, pp. 51–7.

Popper, Karl R. (1957). The Propensity Interpretation of the Calculus of Probability and the Quantum Theory. *The Colston Papers* 9. Ed. by S. Körner, pp. 65–70.

Predd, J. et al. (2009). Probabilistic Coherence and Proper Scoring Rules. *IEEE Transactions on Information Theory* 55, pp. 4786–92.

Pryor, James (2004). What's Wrong with Moore's Argument? *Philosophical Issues* 14, pp. 349–78.

Quinn, Warren S. (1990). The Puzzle of the Self-Torturer. *Philosophical Studies* 59, pp. 79–90.

Ramsey, Frank P. (1929/1990). General Propositions and Causality. In: *Philosophical Papers*. Ed. by D.H. Mellor. Cambridge: Cambridge University Press, pp. 145–63.

Ramsey, Frank P. (1931). Truth and Probability. In: *The Foundations of Mathematics and other Logic Essays*. Ed. by R.B. Braithwaite. New York: Harcourt, Brace and Company, pp. 156–98.

Rees, Martin (2000). *Just Six Numbers: The Deep Forces that Shape the Universe*. New York: Basic Books.

Reichenbach, Hans (1935/1949). *The Theory of Probability.* English expanded version of the German original. Berkeley: University of California Press.

Reichenbach, Hans (1938). *Experience and Prediction.* Chicago: University of Chicago Press.

Reichenbach, Hans (1956). The Principle of Common Cause. In: *The Direction of Time.* Berkeley: University of California Press, pp. 157–60.

Reiss, Julian and Jan Sprenger (2017). Scientific Objectivity. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Winter 2017. Metaphysics Research Lab, Stanford University. URL: https://plato.stanford.edu/archives/win2017/entries/scientific-objectivity/.

Renyi, Alfred (1970). *Foundations of Probability.* San Francisco: Holden-Day.

Resnik, Michael D. (1987). *Choices: An Introduction to Decision Theory.* Minneapolis: University of Minnesota Press.

Roche, William (2014). Evidence of Evidence Is Evidence under Screening-off. *Episteme* 11, pp. 119–24.

Roeper, P. and H. Leblanc (1999). *Probability Theory and Probability Logic.* Toronto: University of Toronto Press.

Rosenkrantz, Roger (1981). *Foundations and Applications of Inductive Probability.* Atascadero, CA: Ridgeview Press.

Rosenthal, Jeffrey S. (2009). A Mathematical Analysis of the Sleeping Beauty Problem. *The Mathematical Intelligencer* 31, pp. 32–37.

Royall, Richard M. (1997). *Statistical Evidence: A Likelihood Paradigm.* New York: Chapman & Hall/CRC.

Salmon, Wesley (1966). *The Foundations of Scientific Inference.* Pittsburgh: University of Pittsburgh Press.

Salmon, Wesley (1975). Confirmation and Relevance. In: *Induction, Probability, and Confirmation.* Ed. by Grover Maxwell and Robert M. Jr. Anderson. Vol. VI. Minnesota Studies in the Philosophy of Science. Minneapolis: University of Minnesota Press, pp. 3–36.

Savage, Leonard J. (1954). *The Foundations of Statistics.* New York: Wiley.

Savage, Leonard J. (1967). Difficulties in the Theory of Personal Probability. *Philosophy of Science* 34, pp. 305–10.

Schervish, M.J., T. Seidenfeld, and J.B. Kadane (2004). Stopping to Reflect. *Journal of Philosophy* 101, pp. 315–22.

Schervish, M.J., T. Seidenfeld, and J.B. Kadane (2009). Proper Scoring Rules, Dominated Forecasts, and Coherence. *Decision Analysis* 6, pp. 202–21.

Schick, Frederic (1986). Dutch Bookies and Money Pumps. *The Journal of Philosophy* 83, pp. 112–19.

Schoenfield, Miriam (2017). The Accuracy and Rationality of Imprecise Credences. *Noûs* 51, pp. 667–85.

Schwarz, Wolfgang (2010). *Lewis on Updating and Self-location.* Blog post available at URL: https://www.umsu.de/wo/2010/563.

Schwarz, Wolfgang (2018). Subjunctive Conditional Probability. *Journal of Philosophical Logic* 47, pp. 47–66.

Scott, Dana (1964). Measurement Structures and Linear Inequalities. *Journal of Mathematical Psychology* 1, pp. 233–47.

Seelig, C. (1956). *Albert Einstein: A Documentary Biography.* London: Staples Press.

Seidenfeld, Teddy (1986). Entropy and Uncertainty. *Philosophy of Science* 53, pp. 467–91.

Seidenfeld, Teddy, M.J. Schervish, and J.B. Kadane (2017). Non-Conglomerability for Countably Additive Measures That Are Not κ-Additive. *The Review of Symbolic Logic* 10, pp. 284–300.

Seidenfeld, Teddy and Larry Wasserman (1993). Dilation for Sets of Probabilities. *The Annals of Statistics* 21, pp. 1139–54.

Selvin, Steve (1975). A Problem in Probability. *The American Statistician* 29. Published among the Letters to the Editor, p. 67.

Shafer, Glenn (1976). *A Mathematical Theory of Evidence*. Princeton, NJ: Princeton University Press.

Shafer, Glenn (1981). Constructive Probability. *Synthese* 48, pp. 1–60.

Shapiro, Amram, Louise Firth Campbell, and Rosalind Wright (2014). *The Book of Odds*. New York: Harper Collins.

Shimony, Abner (1955). Coherence and the Axioms of Confirmation. *Journal of Symbolic Logic* 20, pp. 1–28.

Shimony, Abner (1988). An Adamite Derivation of the Calculus of Probability. In: *Probability and Causality*. Ed. by J.H. Fetzer. Dordrecht: Reidel, pp. 151–61.

Shogenji, Tomoji (2003). A Condition for Transitivity in Probabilistic Support. *British Journal for the Philosophy of Science* 54, pp. 613–16.

Shogenji, Tomoji (2012). The Degree of Epistemic Justification and the Conjunction Fallacy. *Synthese* 184, pp. 29–48.

Shope, R.K. (1978). The Conditional Fallacy in Contemporary Philosophy. *Journal of Philosophy* 75, pp. 397–413.

Shortliffe, E. and B. Buchanan (1975). A Model of Inexact Reasoning in Medicine. *Mathematical Biosciences* 23, pp. 351–79.

Simpson, E.H. (1951). The Interpretation of Interaction in Contingency Tables. *Journal of the Royal Statistical Society, Series B* 13, pp. 238–41.

Skipper, Mattias and Jens Christian Bjerring (2022). Bayesianism for Nonideal Agents. *Erkenntnis* 87, pp. 93–115.

Skyrms, Brian (1980a). *Causal Necessity: A Pragmatic Investigation of the Necessity of Laws*. New Haven CT: Yale University Press.

Skyrms, Brian (1980b). Higher Order Degrees of Belief. In: *Prospects for Pragmatism*. Ed. by D.H. Mellor. Cambridge: Cambridge University Press, pp. 109–37.

Skyrms, Brian (1983). Three Ways to Give a Probability Function a Memory. In: *Testing Scientific Theories*. Ed. by John Earman. Vol. 10. Minnesota Studies in the Philosophy of Science. Minneapolis: University of Minnesota Press, pp. 157–61.

Skyrms, Brian (1987a). Coherence. In: *Scientific Inquiry in Philosophical Perspective*. Ed. by N. Rescher. Pittsburgh: University of Pittsburgh Press, pp. 225–42.

Skyrms, Brian (1987b). Dynamic Coherence and Probability Kinematics. *Philosophy of Science* 54, pp. 1–20.

Skyrms, Brian (2000). *Choice & Chance: An Introduction to Inductive Logic*. 4th edition. Stamford, CT: Wadsworth.

Smith, C.A.B. (1961). Consistency in Statistical Inference and Decision. *Journal of the Royal Statistical Society, Series B* 23, pp. 1–25.

Smithies, Declan (2015). Ideal Rationality and Logical Omniscience. *Synthese* 192, pp. 2769–93.

Sober, Elliott (2008). *Evidence and Evolution*. Cambridge: Cambridge University Press.

Spohn, Wolfgang (2012). *The Laws of Belief: Ranking Theory and its Philosophical Applications*. Oxford: Oxford University Press.

Staffel, Julia (2019). *Unsettled Thoughts: Reasoning and Uncertainty in Epistemology*. Oxford: Oxford University Press.

Stalnaker, Robert C. (1972/1981). Letter to David Lewis. In: *Ifs: Conditionals, Belief, Decision, Chance, and Time*. Ed. by W. Harper, Robert C. Stalnaker, and G. Pearce. Dordrecht: Reidel, pp. 151–2.

Stalnaker, Robert C. (2008). *Our Knowledge of the Internal World.* Oxford: Oxford University Press.

Stalnaker, Robert C. (2011). Responses to Stoljar, Weatherson and Boghossian. *Philosophical Studies* 155, pp. 467–79.

Stefánsson, H. Orri (2017). What Is "Real" in Probabilism? *Australasian Journal of Philosophy* 95, pp. 573–87.

Stefánsson, H. Orri (2018). On the Ratio Challenge for Comparativism. *Australasian Journal of Philosophy* 96, pp. 380–90.

Stephenson, Todd A. (2000). *An Introduction to Bayesian Network Theory and Usage.* Tech. rep. 03. IDIAP.

Sturgeon, Scott (2010). Confidence and Coarse-grained Attitudes. In: *Oxford Studies in Epistemology.* Ed. by Tamar Szabó Gendler and John Hawthorne. Vol. 3. Oxford: Oxford University Press, pp. 126–49.

Suppes, Patrick (1955). The Role of Subjective Probability and Utility in Decision-making. *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability,* pp. 61–73.

Suppes, Patrick (1969). *Studies in the Methodology and Foundations of Science.* Berlin: Springer.

Suppes, Patrick (1974). *Probabilistic Metaphysics.* Uppsala: University of Uppsala Press.

Suppes, Patrick (2002). *Representation and Invariance of Scientific Structures.* Stanford, CA: CSLI Publications.

Tal, Eyal and Juan Comesaña (2017). Is Evidence of Evidence Evidence? *Noûs* 51, pp. 95–112.

Talbott, William J. (1991). Two Principles of Bayesian Epistemology. *Philosophical Studies* 62, pp. 135–50.

Talbott, William J. (2005). Review of "Putting Logic in its Place: Formal Constraints on Rational Belief". *Notre Dame Philosophical Reviews.* URL: http://ndpr.nd.edu/.

Talbott, William J. (2016). Bayesian Epistemology. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Winter 2016. Metaphysics Research Lab, Stanford University. URL: https://plato.stanford.edu/archives/win2016/entries/epistemology-bayesian/.

Teller, Paul (1973). Conditionalization and Observation. *Synthese* 26, pp. 218–58.

Tennant, Neil (2017). Logicism and Neologicism. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Winter 2017. Metaphysics Research Lab, Stanford University. URL: https://plato.stanford.edu/archives/win2017/entries/logicism/.

Tentori, Katya, Vincenzo Crupi, and Selena Russo (2013). On the Determinants of the Conjunction Fallacy: Probability versus Inductive Confirmation. *Journal of Experimental Psychology: General* 142, pp. 235–55.

Titelbaum, Michael G. (2008). The Relevance of Self-locating Beliefs. *Philosophical Review* 117, pp. 555–605.

Titelbaum, Michael G. (2010). Not Enough There There: Evidence, Reasons, and Language Independence. *Philosophical Perspectives* 24, pp. 477–528.

Titelbaum, Michael G. (2013a). *Quitting Certainties: A Bayesian Framework Modeling Degrees of Belief.* Oxford: Oxford University Press.

Titelbaum, Michael G. (2013b). Ten Reasons to Care about the Sleeping Beauty Problem. *Philosophy Compass* 8, pp. 1003–17.

Titelbaum, Michael G. (2015a). Continuing On. *Canadian Journal of Philosophy* 45, pp. 670–91.

Titelbaum, Michael G. (2015b). Rationality's Fixed Point (Or: In Defense of Right Reason). In: *Oxford Studies in Epistemology.* Ed. by Tamar Szabó Gendler and John Hawthorne. Vol. 5. Oxford University Press, pp. 253–94.

Titelbaum, Michael G. (2016). Self-locating Credences. In: *The Oxford Handbook of Probability and Philosophy.* Ed. by Alan Hájek and Christopher R. Hitchcock. Oxford: Oxford University Press, pp. 666–80.

Titelbaum, Michael G. (2019). Precise Credences. In: *The Open Handbook of Formal Epistemology.* Ed. by Richard Pettigrew and Jonathan Weisberg. The PhilPapers Foundation, pp. 1–56.

Tversky, Amos and Daniel Kahneman (1974). Judgment under Uncertainty: Heuristics and Biases. *Science* 185, pp. 1124–31.

Tversky, Amos and Daniel Kahneman (1983). Extensional Versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment. *Psychological Review* 90, pp. 293–315.

Tversky, Amos and Daniel Kahneman (1992). Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty* 5, pp. 297–323.

Tversky, Amos and Derek J. Koehler (1994). Support Theory: A Nonextensional Representation of Subjective Probability. *Psychological Review* 101, pp. 547–67.

van Enk, Steven J. (2015). Betting, Risk, and the Law of Likelihood. *Ergo* 2, pp. 105–21.

van Fraassen, Bas C. (1980). Rational Belief and Probability Kinematics. *Philosophy of Science* 47, pp. 165–87.

van Fraassen, Bas C. (1981). A Problem for Relative Information Minimizers. *British Journal for the Philosophy of Science* 32, pp. 375–79.

van Fraassen, Bas C. (1982). Rational Belief and the Common Cause Principle. In: *What? Where? When? Why?* Ed. by Robert McLaughlin. Dordrecht: Reidel, pp. 193–209.

van Fraassen, Bas C. (1983). Calibration: A Frequency Justification for Personal Probability. In: *Physics Philosophy and Psychoanalysis.* Ed. by R. Cohen and L. Laudan. Dordrecht: Reidel, pp. 295–319.

van Fraassen, Bas C. (1984). Belief and the Will. *The Journal of Philosophy* 81, pp. 235–56.

van Fraassen, Bas C. (1989). *Laws and Symmetry.* Oxford: Clarendon Press.

van Fraassen, Bas C. (1995). Belief and the Problem of Ulysses and the Sirens. *Philosophical Studies* 77, pp. 7–37.

van Fraassen, Bas C. (1999). Conditionalization: A New Argument For. *Topoi* 18, pp. 93–6.

van Fraassen, Bas C. (2005). Conditionalizing on Violated Bell's Inequalities. *Analysis* 65, pp. 27–32.

Van Horn, Kevin S. (2003). Constructing a Logic of Plausible Inference: A Guide to Cox's Theorem. *International Journal of Approximate Reasoning* 34, pp. 3–24.

Venn, John (1866). *The Logic of Chance.* London, Cambridge: Macmillan.

Vineberg, Susan (2011). Dutch Book Arguments. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Summer 2011. URL: https://plato.stanford.edu/archives/su2011/entries/dutch-book/.

von Mises, Richard (1928/1957). *Probability, Statistics and Truth.* (English edition of the original German *Wahrscheinlichkeit, Statistik und Wahrheit.*) New York: Dover.

von Neumann, J. and O. Morgenstern (1947). *Theory of Games and Economic Behavior.* 2nd edition. Princeton, NJ: Princeton University Press.

Vranas, Peter B.M. (2004). Hempel's Raven Paradox: A Lacuna in the Standard Bayesian Solution. *British Journal for the Philosophy of Science* 55, pp. 545–60.

Wainer, Howard (2011). *Uneducated Guesses: Using Evidence to Uncover Misguided Education Policies.* Princeton, NJ: Princeton University Press.

Walley, Peter (1991). *Statistical Reasoning with Imprecise Probabilities.* London: Chapman and Hall.

Wasserstein, Ronald L. and Nicole A. Lazar (2016). The ASA's Statement on p-values: Context, Process, and Purpose. *The American Statistician.* DOI: 10.1080/00031305.2016.1154108.

Weatherson, Brian (2002). Keynes, Uncertainty and Interest Rates. *Cambridge Journal of Economics* 26, pp. 47–62.

Weatherson, Brian (2011). Stalnaker on Sleeping Beauty. *Philosophical Studies* 155, pp. 445–56.

Weatherson, Brian (2015). For Bayesians, Rational Modesty Requires Imprecision. *Ergo* 2, pp. 529–45.

Weatherson, Brian and Andy Egan (2011). Epistemic Modals and Epistemic Modality. In: *Epistemic Modality.* Ed. by Andy Egan and Brian Weatherson. Oxford: Oxford University Press, pp. 1–18.

Weintraub, Ruth (2001). The Lottery: A Paradox Regained and Resolved. *Synthese* 129, pp. 439–49.

Weirich, Paul (2012). Causal Decision Theory. In: *The Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Winter 2012. URL: https://plato.stanford.edu/archives/win2012/entries/decision-causal/.

Weisberg, Jonathan (2007). Conditionalization, Reflection, and Self-Knowledge. *Philosophical Studies* 135, pp. 179–97.

Weisberg, Jonathan (2009). Varieties of Bayesianism. In: *Handbook of the History of Logic.* Ed. by Dov. M Gabbya, Stephan Hartmann, and John Woods. Vol. 10: Inductive Logic. Oxford: Elsevier.

White, Roger (2005). Epistemic Permissiveness. *Philosophical Perspectives* 19, pp. 445–59.

White, Roger (2006). Problems for Dogmatism. *Philosophical Studies* 131, pp. 525–57.

White, Roger (2010). Evidential Symmetry and Mushy Credence. In: *Oxford Studies in Epistemology.* Ed. by Tamar Szabó Gendler and John Hawthorne. Vol. 3. Oxford: Oxford University Press, pp. 161–86.

Williams, J. Robert G. (ms). A Non-Pragmatic Dominance Argument for Conditionalization. Unpublished manuscript.

Williams, J. Robert G. (2016). Probability and Non-Classical Logic. In: *Oxford Handbook of Probability and Philosophy.* Ed. by Alan Hájek and Christopher R. Hitchcock. Oxford: Oxford University Press.

Williamson, Timothy (2000). *Knowledge and its Limits.* Oxford: Oxford University Press.

Williamson, Timothy (2007). How Probable Is an Infinite Sequence of Heads? *Analysis* 67, pp. 173–80.

Wittgenstein, Ludwig (1921/1961). *Tractatus Logico-Philosophicus.* Translated by D.F. Pears and B.F. McGuinness. London: Routledge.

Wong, S.M. et al. (1991). Axiomatization of Qualitative Belief Structure. *IEEE Transactions on Systems, Man and Cybernetics* 21, pp. 726–34.

Woolston, Chris (2015). Psychology Journal Bans *P* Values. *Nature* 519.9. URL: https://doi.org/10.1038/519009f.

Wroński, Leszek and Godziszewski, Michał Tomasz (2017). *The Stubborn Non-probabilist—'Negation Incoherence' and a New Way to Block the Dutch Book Argument.* International Workshop on Logic, Rationality and Interaction. Berlin, Heidelberg: Springer, pp. 256–67.

Yalcin, Seth (2012). A Counterexample to Modus Tollens. *Journal of Philosophical Logic* 41, pp. 1001–24.

Yule, G.U. (1903). Notes on the Theory of Association of Attributes in Statistics. *Biometrika* 2, pp. 121–34.

Zynda, Lyle (1995). Old Evidence and New Theories. *Philosophical Studies* 77, pp. 67–95.

Zynda, Lyle (2000). Representation Theorems and Realism about Degrees of Belief. *Philosophy of Science* 67, pp. 45–69.