

# The Frege Reader

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*Edited by*  
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# Abbreviations of Works by Frege

The following abbreviations are used for works by Frege referred to in the editorial material. Full citations are provided in the Bibliography, where a complete list of Frege's works, chronologically ordered, can be found. Where a date is set in square brackets in column two, the work concerned is listed in the Bibliography under the first date given in column three.

<i>ACN</i>	1882	'On the Aim of the "Conceptual Notation"'
<i>APCN</i>	1879	'Applications of the "Conceptual Notation"'
<i>BLA</i>	[1964]	<i>The Basic Laws of Arithmetic</i> (tr. of part of <i>GG</i> , 1893, 1903, by M. Furth)
<i>BLC</i>	1880/81	'Boole's Logical Calculus and the Concept-script'
<i>BLF</i>	1882	'Boole's Logical Formula-language and my Concept-script'
<i>BS</i>	1879	<i>Begriffsschrift</i>
<i>BSA</i>	1964	<i>Begriffsschrift und andere Aufsätze</i> (ed. I. Angelelli)
<i>BSLD</i>	1906	'A Brief Survey of my Logical Doctrines'
<i>CES</i>	1895	'A Critical Elucidation of Some Points in E. Schröder . . .'
<i>CN</i>	1972	<i>Conceptual Notation</i> (tr. of <i>BS</i> by T. W. Bynum)
<i>CO</i>	1892	'On Concept and Object'
<i>CP</i>	[1984]	<i>Collected Papers</i> (tr. by M. Black et al. of <i>KS</i> , 1967)
<i>CSB</i>	1892	'[Comments on <i>Sinn</i> and <i>Bedeutung</i> ']
<i>CT</i>	1923	'Compound Thoughts'
<i>DPE</i>	before 1884	'[Dialogue with Pünjer on Existence]'
<i>EG</i>	1899–1906?	'On Euclidean Geometry'
<i>FA</i>	[1950]	<i>The Foundations of Arithmetic</i> (tr. of <i>GL</i> , 1884, by J. L. Austin)
<i>FC</i>	1891	'Function and Concept'
<i>FG</i>	1971	<i>On the Foundations of Geometry</i> (ed. and tr. by E.-H. W. Kluge)

<i>FGI</i>	1903	'On the Foundations of Geometry: First Series'
<i>FGII</i>	1906	'On the Foundations of Geometry: Second Series'
<i>FR</i>	1997	<i>The Frege Reader</i> (ed. M. Beaney)
<i>GG</i>	1893/1903	<i>Grundgesetze der Arithmetik</i> , Vol. I, 1893; Vol. II, 1903
<i>GL</i>	1884	<i>Die Grundlagen der Arithmetik</i>
<i>GR</i>	1873	'On a Geometrical Representation of Imaginary Forms in the Plane'
<i>IL</i>	1906	'Introduction to Logic'
<i>KS</i>	1967	<i>Kleine Schriften</i> (tr. by M. Black et al. as <i>CP</i> )
<i>LI</i>	1977	<i>Logical Investigations</i> (now in <i>CP</i> )
<i>LM</i>	1914	'Logic in Mathematics'
<i>MBLI</i>	1915	'My Basic Logical Insights'
<i>MC</i>	1874	'Methods of Calculation . . .'
<i>N</i>	1918	'Negation'
<i>NLD</i>	1919	'[Notes for Ludwig Darmstaedter]'
<i>NS</i>	1969	<i>Nachgelassene Schriften</i> (tr. by P. Long and R. White as <i>PW</i> )
<i>OCN</i>	1891/92	'On the Concept of Number'
<i>PCN</i>	1897	'On Mr. Peano's Conceptual Notation and My Own'
<i>PMC</i>	[1980]	<i>Philosophical and Mathematical Correspondence</i> (tr. by H. Kaal of <i>WB</i> , 1976)
<i>PW</i>	[1979]	<i>Posthumous Writings</i> (tr. by P. Long and R. White of <i>NS</i> , 1969)
<i>PWLB</i>	1897	'Logic'
<i>RH</i>	1894	'Review of E. G. Husserl, <i>Philosophie der Arithmetik I</i> '
<i>SB</i>	1892	'On <i>Sinn</i> and <i>Bedeutung</i> '
<i>SJCN</i>	1882	'On the Scientific Justification of a Conceptual Notation'
<i>SKM</i>	1924/25	'Sources of Knowledge of Mathematics . . .'
<i>T</i>	1918	'Thought'
<i>TPW</i>	1952	<i>Translations from the Philosophical Writings of Gottlob Frege</i>
<i>WB</i>	1976	<i>Wissenschaftlicher Briefwechsel</i> (tr. by H. Kaal as <i>PMC</i> )

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# Glossary

The various key terms used by Frege have been translated in the present volume as follows (unless otherwise indicated in the text):

<i>anduten</i> <sup>1</sup>	indicate
<i>Anschauung</i>	intuition
<i>Anzahl</i> <sup>2</sup>	Number
<i>Art des Gegebenseins</i>	mode of presentation
<i>bedeuten</i> <sup>3</sup>	mean, signify [in his early work] refer to, stand for [in his later work]
<i>Bedeutung</i> <sup>3</sup>	meaning, significance [in his early work] <i>Bedeutung</i> [left untranslated in his later work]
<i>Begriff</i> <sup>4</sup>	concept
<i>begrifflicher Inhalt</i> <sup>5</sup>	conceptual content
<i>Begriffsschrift</i> <sup>6</sup>	<i>Begriffsschrift</i> [left untranslated throughout]
<i>Behauptungssatz</i>	assertoric sentence
<i>beiderseits eindeutig</i> <sup>7</sup>	one-one
<i>Bestimmungsweise</i>	mode of determination

<sup>1</sup> This term is applied, in particular, to letters used as variables, which do not mean [*bedeuten*] anything by themselves, but merely indicate (indefinitely) [(unbestimmt) *anduten*]. Cf. e.g. *FC*, p. 133 below; *GG*, I, §1 (p. 211 below).

<sup>2</sup> On the translation of 'Anzahl' by 'Number' (with a capital 'N') and 'Zahl' by 'number', see fn. 6 on p. 91 below.

<sup>3</sup> For discussion of the problems involved in translating 'bedeuten' and 'Bedeutung', see Introduction, §4.

<sup>4</sup> In Frege's later work, a concept is the *Bedeutung* of a concept word (see especially *CSB*, pp. 172-80 below), and is regarded as an 'unsaturated' entity (see especially *CO*, pp. 181-93 below).

<sup>5</sup> The term is used in *BS* (see especially §3; p. 53 below). It is the notion of 'conceptual content' that later bifurcates into the notions of 'Sinn' and 'Bedeutung'.

<sup>6</sup> Literally translated as 'concept-script' or 'conceptual notation', Frege uses the term to refer to his logical symbolism, and it also formed the title of his first book, which introduced this symbolism. It is left untranslated in the present volume; and the context makes clear when Frege is referring to his logical system and when to his first book.

<sup>7</sup> On Frege's use of this term, see fn. 10 on p. 198 below.

<i>beurteilbarer Inhalt</i> <sup>8</sup>	judgeable content
<i>Beziehung</i>	relation
<i>eindeutig</i> <sup>9</sup>	many-one
<i>formal</i>	formal(ist)
<i>Gedanke</i>	thought [understood, in his later work, as the <i>sense</i> expressed by a proposition]
<i>Gegenstand</i>	object
<i>gewöhnliche (Bedeutung)</i> <sup>10</sup>	customary ( <i>Bedeutung</i> )
<i>gleich</i> <sup>11</sup>	equal, identical
<i>Gleichheit</i> <sup>11</sup>	equality, identity
<i>Gleichung</i> <sup>11</sup>	equation
<i>gleichzählig</i>	equinumerous
<i>Inhalt</i>	content
<i>Merkmal</i> <sup>12</sup>	mark
<i>objektiv</i> <sup>13</sup>	objective
<i>Satz</i> <sup>14</sup>	sentence, proposition, theorem, clause
<i>Sinn</i>	sense
<i>Stufe</i>	level (of a concept or function)
<i>Umfang eines Begriffes</i>	extension of a concept
<i>ungerade (Bedeutung)</i> <sup>15</sup>	indirect ( <i>Bedeutung</i> )
<i>ungesättigt</i> <sup>16</sup>	unsaturated
<i>Urteil</i>	judgement

<sup>8</sup> Geach, in *TPW*, translated this term (used in *BS*) as ‘content of possible judgement’. The simpler and more literal ‘judgeable content’ is preferred here.

<sup>9</sup> On Frege’s (logical) definition of a *many-one* relation, see p. 77 below.

<sup>10</sup> On Frege’s distinction between ‘customary’ and ‘indirect’ *Bedeutung*, see *SB*, pp. 154, 159ff. below.

<sup>11</sup> On the translation of ‘gleich’, ‘gleichheit’ and ‘Gleichung’, see fn. 3 on p. 85 below. Frege makes it clear that he treats ‘equality’ (‘Gleichheit’) and ‘identity’ (‘Identität’) as synonymous (see e.g. fn. A on p. 151 below).

<sup>12</sup> On the notion of a ‘mark’, see *GL*, §53 (pp. 102–3 below); *CO*, pp. 189–90 below.

<sup>13</sup> Frege distinguishes what is *objective* (*objektiv*) from what is *actual* (*wirklich*); see *GL*, §26 (cf. p. 96 below). Numbers, for example, according to Frege, are *objective* but not *actual* objects.

<sup>14</sup> Some writers now distinguish between *sentence* (the linguistic expression) and *proposition* (what a sentence *expresses* – what Frege might have called its *sense* or the *thought* it expresses): on this view, two different sentences, e.g. ‘Pegasus ist ein Pferd’ and ‘Pegasus is a horse’, can express the same proposition. The distinction was not drawn by Frege, and is not observed here: ‘sentence’ and ‘proposition’ are to be understood synonymously. ‘Satz’ can also mean ‘theorem’ or just ‘clause’, and these have been used too, according to the context.

<sup>15</sup> Cf. fn. 10 above.

<sup>16</sup> On the ‘unsaturated’ nature of concepts and other functions, see especially *CO*, pp. 181–93 below. In *TPW*, the translated term is kept in quotation marks to signal that it is being used metaphorically. Frege does indeed make clear its metaphorical use, but since he does not so signal it himself, and it is fairly clear that it is metaphorical, the policy has not been followed here.



<i>Verneinung</i>	negation
<i>Vorstellung</i> <sup>17</sup>	idea, image
<i>Wahrheitswert</i>	truth-value [in his later work, regarded as the <i>Bedeutung</i> of a sentence]
<i>Wertverlauf</i> <sup>18</sup>	value-range (of a function)
<i>Zahl</i> <sup>19</sup>	number

<sup>17</sup> By 'Vorstellung', Frege always understands something essentially private and subjective to be sharply distinguished from 'Begriff' ('concept') and 'Gedanke' ('thought').

<sup>18</sup> (On the translation of this term, and Frege's understanding of it, see fn. 2 on p. 135 below.

<sup>19</sup> Cf. fn. 2 above.

# Thought<sup>1</sup>

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[‘Der Gedanke’ was published in *Beiträge zur Philosophie des deutschen Idealismus I* (1918–19), pp. 58–77, as the first part of a series of three papers entitled ‘Logical Investigations’.<sup>2</sup> With ‘Über Sinn und Bedeutung’, it is one of Frege’s two most influential and widely discussed papers.]

41N Just as ‘beautiful’ points the way for aesthetics and ‘good’ for ethics, so do words like ‘true’ for logic. All sciences have truth as their goal; but logic is also concerned with it in a quite different way: logic has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. The word ‘law’ is used in two senses. When we speak of moral or civil laws we mean [*meinen*] prescriptions, which ought to be obeyed but with which actual occurrences are not always in conformity. Laws of nature are general features of what happens in nature, and occurrences in nature are always in accordance with them. It is rather in this sense that I speak of laws of truth. Here of course it is not a matter of what happens but of what is. From the laws of truth there follow prescriptions about asserting, thinking, judging, inferring. And we may very well speak of laws of thought in this way too. But there is at once a danger here of confusing different things. People may very well interpret the expression ‘law of thought’ by analogy with ‘law of nature’ and then have in mind general features of thinking as a mental occurrence. A law of thought in this sense would be a psychological law. And so they might come to believe that logic deals with the mental process of thinking and with the psychological laws in accordance with which this takes place. That would be misunderstanding the task of logic, for truth has not here been given its proper place. Error and superstition have

<sup>1</sup> Translated by Peter Geach and R. H. Stoothoff (*CP*, pp. 351–72/*KS*, pp. 342–62). Page numbers in the margin refer to the original journal in which the paper was published. Unless otherwise indicated, where the verb ‘mean’ appears in this translation, it has been used in rendering certain German constructions involving ‘sollen’ or ‘wollen’.

<sup>2</sup> The other two parts are ‘Die Verneinung’ (‘Negation’; see pp. 346–61 below), and ‘Gedankengefüge’ (‘Compound Thoughts’; *CP*, pp. 390–406/*KS*, pp. 378–94).

59 causes just as much as correct cognition. Whether what you take for true is false | or true, your so taking it comes about in accordance with psychological laws. A derivation from these laws, an explanation of a mental process that ends in taking something to be true, can never take the place of proving what is taken to be true. But may not logical laws also have played a part in this mental process? I do not want to dispute this, but if it is a question of truth this possibility is not enough. For it is also possible that something non-logical played a part in the process and made it swerve from the truth. We can decide only after we have come to know the laws of truth; but then we can probably do without the derivation and explanation of the mental process, if our concern is to decide whether the process terminates in *justifiably* taking something to be true. In order to avoid any misunderstanding and prevent the blurring of the boundary between psychology and logic, I assign to logic the task of discovering the laws of truth, not the laws of taking things to be true or of thinking. The *Bedeutung* of the word 'true' is spelled out in the laws of truth.

But first I shall attempt to outline roughly how I want to use 'true' ['*wahr*'] in this connection, so as to exclude irrelevant uses of the word. 'True' is not to be used here in the sense of 'genuine' ['*wahrhaftig*'] or 'veracious' ['*wahrheitsliebend*']; nor yet in the way it sometimes occurs in discussion of artistic questions, when, for example, people speak of truth in art, when truth is set up as the aim of art, when the truth of a work of art or true feeling is spoken of. Again, the word 'true' is prefixed to another word in order to show that the word is to be understood in its proper, unadulterated sense. This use too lies off the path followed here. What I have in mind is that sort of truth which it is the aim of science to discern.

Grammatically, the word 'true' looks like a word for a property. So we want to delimit more closely the region within which truth can be predicated, the region in which there is any question of truth. We find truth predicated of pictures, ideas, sentences, and thoughts. It is striking that visible and audible things turn up here along with things which cannot be perceived with the senses. This suggests that alterations in sense [*Verschiebungen des Sinnes*] have taken place. So indeed they have! Is a picture considered as a mere visible and tangible thing really true, and a stone or a leaf not true? Obviously we could not call a picture true unless there were an intention involved. A picture is meant to represent something. (Even an idea is not called true in itself, but only with respect to an intention that the idea should correspond to something). It might be supposed from this that truth consists in a correspondence of a picture to what it depicts. Now a correspondence is a relation. But this goes against the use of the word 'true', which is not a relative term and contains no indication of anything else to which something is to

correspond. If I do not know that a picture is meant to represent Cologne Cathedral then I do not know | what to compare the picture with in order to decide on its truth. A correspondence, moreover, can only be perfect if the corresponding things coincide and so just are not different things. It is supposed to be possible to test the genuineness of a bank-note by comparing it stereoscopically with a genuine one. But it would be ridiculous to try to compare a gold piece stereoscopically with a twenty-mark note. It would only be possible to compare an idea with a thing if the thing were an idea too. And then, if the first did correspond perfectly with the second, they would coincide. But this is not at all what people intend when they define truth as the correspondence of an idea with something real. For in this case it is essential precisely that the reality shall be distinct from the idea. But then there can be no complete correspondence, no complete truth. So nothing at all would be true; for what is only half true is untrue. Truth does not admit of more or less.

- But could we not maintain that there is truth when there is correspondence in a certain respect? But which respect? For in that case what ought we to do so as to decide whether something is true? We should have to inquire whether it is *true* that an idea and a reality, say, correspond in the specified respect. And then we should be confronted by a question of the same kind, and the game could begin again. So the attempted explanation of truth as correspondence breaks down. And any other attempt to define truth also breaks down. For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it were *true* that the characteristics were present. So we should be going round in a circle. So it seems likely that the content of the word 'true' is *sui generis* and indefinable.

When we ascribe truth to a picture we do not really mean to ascribe a property which would belong to this picture quite independently of other things; we always have in mind some totally different object and we want to say that the picture corresponds in some way to this object. 'My idea corresponds to Cologne Cathedral' is a sentence, and now it is a matter of the truth of this sentence. So what is improperly called the truth of pictures and ideas is reduced to the truth of sentences. What is it that we call a sentence? A series of sounds, but only if it has a sense (which is not to say that *any* series of sounds that has a sense is a sentence). And when we call a sentence true we really mean [*meinen*] that its sense is true. And hence the only thing that raises the question of truth at all is the sense of sentences. Now is the sense of a sentence an idea? In any case, truth does not consist in correspondence of the sense with something else, for otherwise the question of truth would get reiterated to infinity.

Without offering this as a definition, I call a 'thought' something for

61 which the question of truth can arise at all. So I count what is false | among thoughts no less than what is true.<sup>A</sup> So I can say: thoughts are senses of sentences, without wishing to assert that the sense of every sentence is a thought. The thought, in itself imperceptible by the senses, gets clothed in the perceptible garb of a sentence, and thereby we are enabled to grasp it. We say a sentence *expresses* a thought.

A thought is something imperceptible: anything the senses can perceive is excluded from the realm of things for which the question of truth arises. Truth is not a quality that answers to a particular kind of sense impressions. So it is sharply distinguished from the qualities we call by the names 'red', 'bitter', 'lilac-smelling'. But do we not see that the Sun has risen? And do we not then also see that this is true? That the Sun has risen is not an object emitting rays that reach my eyes; it is not a visible thing like the Sun itself. That the Sun has risen is recognized to be true on the basis of sense impressions. But being true is not a sensible, perceptible, property. A thing's being magnetic is also recognized on the basis of sense impressions of the thing, although this property does not answer, any more than truth does, to a particular kind of sense impressions. So far these properties agree. However, we do need sense impressions in order to recognize a body as magnetic. On the other hand, when I find it is true that I do not smell anything at this moment, I do not do so on the basis of sense impressions.

All the same it is something worth thinking about that we cannot recognize a property of a thing without at the same time finding the thought *this thing has this property* to be true. So with every property of a thing there is tied up a property of a thought, namely truth. It is also worth noticing that the sentence 'I smell the scent of violets' has just the same content as the sentence 'It is true that I smell the scent of violets'.<sup>3</sup> So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth. And yet is it not a great result when the scientist after much hesitation and laborious researches can finally say 'My conjecture is true'? The *Bedeutung* of the word 'true' seems to be altogether *sui generis*. May we not be dealing here with something which cannot be called a property in the ordinary sense at all? In spite

<sup>A</sup> So, similarly, people have said 'a judgement is something which is either true or false'. In fact I use the word 'thought' more or less in the sense 'judgement' has in the writings of logicians. I hope it will become clear in what follows why I choose 'thought'. Such an explanation has been objected to on the ground that it makes a division of judgements into true and false judgements – perhaps the least significant of all possible divisions among judgements. But I cannot see that it is a logical fault that a division is given along with the explanation. As for the division's being significant, we shall perhaps find we must hold it in no small esteem, if, as I have said, it is the word 'true' that points the way for logic.

<sup>3</sup> Cf. II., p. 297 above; *MBLL*, p. 323 above.

of this doubt I will begin by expressing myself in accordance with ordinary usage, | as if truth were a property, until some more appropriate way of speaking is found.

In order to bring out more precisely what I want to call 'thought', I shall distinguish various kinds of sentences.<sup>B</sup> We should not wish to deny sense to a command, but this sense is not such that the question of truth could arise for it. Therefore I shall not call the sense of a command a thought. Sentences expressing wishes or requests are ruled out in the same way. Only those sentences in which we communicate or assert something come into the question. But here I do not count exclamations in which one vents one's feelings, groans, sighs, laughs – unless it has been decided by some special convention that they are to communicate something. But how about interrogative sentences? In a word-question<sup>4</sup> we utter an incomplete sentence, which is meant to be given a true sense just by means of the completion for which we are asking. Word-questions are accordingly left out of consideration here. Propositional questions<sup>5</sup> are a different matter. We expect to hear 'yes' or 'no'. The answer 'yes' means [*besagt*] the same as an assertoric sentence, for in saying 'yes' the speaker presents as true the thought that was already completely contained in the interrogative sentence. This is how a propositional question can be formed from any assertoric sentence. And this is why an exclamation cannot be regarded as a communication: no corresponding propositional question can be formed. An interrogative sentence and an assertoric one contain the same thought; but the assertoric sentence contains something else as well, namely assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an assertoric sentence: the content, which it has in common with the corresponding propositional question; and assertion. The former is the thought or at least contains the thought. So it is possible to express a thought without laying it down as true. The two things are so closely joined in an assertoric sentence that it is easy to overlook their separability. Consequently we distinguish:

- (1) the grasp of a thought – thinking,
- (2) the acknowledgement of the truth of a thought – the act of judgement,<sup>C</sup>
- (3) the manifestation of this judgement – assertion.

<sup>B</sup> I am not using the word 'sentence' [*Satz*'] here in quite the same sense as grammar does, which also includes subordinate clauses. An isolated subordinate clause does not always have a sense about which the question of truth can arise, whereas the complex sentence to which it belongs has such a sense. [In the present volume, 'Satz' has alternatively been translated as 'proposition'. Cf. Glossary above.]

<sup>C</sup> It seems to me that thought and judgement have not hitherto been adequately distin-

<sup>4</sup> Frege means a question introduced by an interrogative word like 'who?' (*Trs.*)

<sup>5</sup> I.e. yes-no questions: the German is 'Satzfragen'. (*Trs.*)

63 We have already performed the first act when we form a propositional question. An advance in science usually takes place in this way: first a thought is grasped, and thus may perhaps be expressed in a propositional question; after appropriate investigations, this thought is finally recognized to be true. | We express acknowledgement of truth in the form of an assertoric sentence. We do not need the word 'true' for this. And even when we do use it the properly assertoric force does not lie in it, but in the assertoric sentence-form; and where this form loses its assertoric force the word 'true' cannot put it back again. This happens when we are not speaking seriously. As stage thunder is only sham thunder and a stage fight only a sham fight, so stage assertion is only sham assertion. It is only acting, only fiction. When playing his part the actor is not asserting anything; nor is he lying, even if he says something of whose falsehood he is convinced. In poetry we have the case of thoughts being expressed without being actually put forward as true, in spite of the assertoric form of the sentence; although the poem may suggest to the hearer that he himself should make an assenting judgement. Therefore the question still arises, even about what is presented in the assertoric sentence-form, whether it really contains an assertion. And this question must be answered in the negative if the requisite seriousness is lacking. It is unimportant whether the word 'true' is used here. This explains why it is that nothing seems to be added to a thought by attributing to it the property of truth.

An assertoric sentence often contains, over and above a thought and assertion, a third component not covered by the assertion. This is often meant to act on the feelings and mood of the hearer, or to arouse his imagination. Words like 'regrettably' [*leider*] and 'fortunately' [*gottlob*] belong here. Such constituents of sentences are more strongly prominent in poetry, but are seldom wholly absent from prose. They occur more rarely in mathematical, physical, or chemical expositions than in historical ones. What are called the humanities are closer to poetry, and are therefore less scientific, than the exact sciences, which are drier in proportion to being more exact; for exact science is directed toward truth and truth alone. Therefore all constituents of sentences not covered by the assertoric force do not belong to scientific exposition; but they are sometimes hard to avoid, even for one who sees the danger connected with them. Where the main thing is to approach by way of

guished. Perhaps language is misleading. For we have no particular bit of assertoric sentences which corresponds to assertion; that something is being asserted is implicit rather in the assertoric form. We have the advantage in German that main and subordinate clauses are distinguished by the word-order. However in this connection we must observe that a subordinate clause may also contain an assertion, and that often neither main nor subordinate clause expresses a complete thought by itself but only the complex sentence does.

intimation what cannot be conceptually grasped, these constituents are fully justified. The more rigorously scientific an exposition is, the less the nationality of its author will be discernible and the easier it will be to translate. On the other hand, the constituents of language to which I here want to call attention make the translation of poetry very difficult, indeed make perfect translation almost always impossible, for it is just in what largely makes the poetic value that languages most differ.

It makes no difference to the thought whether I use the word 'horse' or 'steed' or 'nag' or 'prad'.<sup>6</sup> The assertoric force does not cover the ways in which these words differ. What is called mood, atmosphere, illumination in a poem, what is portrayed by intonation and rhythm, does not belong to the thought. |

6-1

Much in language serves to aid the hearer's understanding, for instance emphasizing part of a sentence by stress or word-order. Here let us bear in mind words like 'still' and 'already'. Somebody using the sentence 'Alfred has still not come' actually says 'Alfred has not come', and at the same time hints – but only hints – that Alfred's arrival is expected. Nobody can say: Since Alfred's arrival is not expected, the sense of the sentence is false. The way that 'but' differs from 'and' is that we use it to intimate that what follows it contrasts with what was to be expected from what preceded it. Such conversational suggestions make no difference to the thought. A sentence can be transformed by changing the verb from active to passive and at the same time making the accusative into the subject. In the same way we may change the dative into the nominative and at the same time replace 'give' by 'receive'. Naturally such transformations are not trivial in every respect; but they do not touch the thought, they do not touch what is true or false. If the inadmissibility of such transformations were recognized as a principle, then any profound logical investigation would be hindered.<sup>7</sup> It is just as important to ignore distinctions that do not touch the heart of the matter, as to make distinctions which concern essentials. But what is essential depends on one's purpose. To a mind concerned with the beauties of language, what is trivial to the logician may seem to be just what is important.

Thus the content of a sentence often goes beyond the thought expressed by it. But the opposite often happens too; the mere wording, which can be made permanent by writing or the gramophone, does not suffice for the expression of the thought. The present tense is used in two ways: first, in order to indicate a time; second, in order to eliminate any temporal restriction, where timelessness or eternity is part of the thought – consider for instance the laws of mathematics. Which of

<sup>6</sup> The German words here are 'Pferd', 'Roß', 'Gaul' and 'Mähre'.

<sup>7</sup> Cf. *CO*, fn. G, pp. 184 5 above; *PWLB*, pp. 240 4 above.



the two cases occurs is not expressed but must be divined. If a time-indication is conveyed by the present tense one must know when the sentence was uttered in order to grasp the thought correctly. Therefore the time of utterance is part of the expression of the thought. If someone wants to say today what he expressed yesterday using the word 'today', he will replace this word with 'yesterday'. Although the thought is the same its verbal expression must be different in order that the change of sense which would otherwise be effected by the differing times of utterance may be cancelled out. The case is the same with words like 'here' and 'there'. In all such cases the mere wording, as it can be preserved in writing, is not the complete expression of the thought; the knowledge of certain conditions accompanying the utterance, which are used as means of expressing the thought, is needed for us to grasp the thought correctly. Pointing the finger, hand gestures, glances may belong here too. The same utterance containing the word 'I' in the mouths of different men will express different thoughts of which some may be true, others false.<sup>8</sup> |

65 The occurrence of the word 'I' in a sentence gives rise to some further questions.

Consider the following case. Dr Gustav Lauben says, 'I was wounded'. Leo Peter hears this and remarks some days later, 'Dr Gustav Lauben was wounded'. Does this sentence express the same thought as the one Dr Lauben uttered himself? Suppose that Rudolph Lingens was present when Dr Lauben spoke and now hears what is related by Leo Peter. If the same thought was uttered by Dr Lauben and Leo Peter, then Rudolph Lingens, who is fully master of the language and remembers what Dr Lauben said in his presence, must now know at once from Leo Peter's report that he is speaking of the same thing. But knowledge of the language is a special thing when proper names are involved. It may well be the case that only a few people associate a definite thought with the sentence 'Dr Lauben was wounded'. For complete understanding one needs in this case to know the expression 'Dr Gustav Lauben'. Now if both Leo Peter and Rudolph Lingens understand by 'Dr Gustav Lauben' the doctor who is the only doctor living in a house known to both of them, then they both understand the sentence 'Dr Gustav Lauben was wounded' in the same way; they associate the same thought with it. But it is also possible that Rudolph Lingens does not know Dr Lauben personally and does not know that it was Dr Lauben who recently said 'I was wounded'. In this case Rudolph Lingens cannot know that the same affair is in question. I say, therefore, in this case:

<sup>8</sup> This and the following five paragraphs, concerning thoughts expressed with the use of indexicals, have generated much controversy. For discussion of some of the problems raised, see the Introduction, pp. 31-5 above.

the thought which Leo Peter expresses is not the same as that which Dr Lauben uttered.

Suppose further that Herbert Garner knows that Dr Gustav Lauben was born on 13 September 1875 in N.N. and this is not true of anyone else; suppose, however, that he does not know where Dr Lauben now lives nor indeed anything else about him. On the other hand, suppose Leo Peter does not know that Dr Lauben was born on 13 September 1875 in N.N. Then as far as the proper name 'Dr Gustav Lauben' is concerned, Herbert Garner and Leo Peter do not speak the same language, although they do in fact designate the same man with this name; for they do not know that they are doing so. Therefore Herbert Garner does not associate the same thought with the sentence 'Dr Gustav Lauben was wounded' as Leo Peter wants to express with it. To avoid the awkwardness that Herbert Garner and Leo Peter are not speaking the same language, I shall suppose that Leo Peter uses the proper name 'Dr Lauben' and Herbert Garner uses the proper name 'Gustav Lauben'. Then it is possible that Herbert Garner takes the sense of the sentence 'Dr Lauben was wounded' to be true but is misled by false information into taking the sense of the sentence 'Gustav Lauben was wounded' to be false. So given our assumptions these thoughts are different.

Accordingly, with a proper name, it is a matter of the way that the object so designated is presented. This may happen in different ways, and | to every such way there corresponds a special sense of a sentence containing the proper name. The different thoughts thus obtained from the same sentences correspond in truth-value, of course; that is to say, if one is true then all are true, and if one is false then all are false. Nevertheless the difference must be recognized. So we must really stipulate that for every proper name there shall be just one associated manner of presentation of the object so designated. It is often unimportant that this stipulation should be fulfilled, but not always.

Now everyone is presented to himself in a special and primitive way, in which he is presented to no one else. So, when Dr Lauben has the thought that he was wounded, he will probably be basing it on this primitive way in which he is presented to himself. And only Dr Lauben himself can grasp thoughts specified in this way. But now he may want to communicate with others. He cannot communicate a thought he alone can grasp. Therefore, if he now says 'I was wounded', he must use 'I' in a sense which can be grasped by others, perhaps in the sense of 'he who is speaking to you at this moment'; by doing this he makes the conditions accompanying his utterance serve towards the expression of a thought.<sup>1)</sup>

<sup>1)</sup> I am not here in the happy position of a mineralogist who shows his audience a rock-crystal: I cannot put a thought in the hands of my readers with the request that they

Yet there is a doubt. Is it at all the same thought which first this and then that man expresses?

A man who is still unaffected by philosophy first of all gets to know things he can see and touch, can in short perceive with the senses, such as trees, stones and houses, and he is convinced that someone else can equally see and touch the same tree and the same stone as he himself sees and touches. Obviously a thought does not belong with these things. Now can it, nevertheless, like a tree be presented to people as the same?

Even an unphilosophical man soon finds it necessary to recognize an inner world distinct from the outer world, a world of sense impressions, of creations of his imagination, of sensations, of feelings and moods, a world of inclinations, wishes and decisions. For brevity's sake I want to use the word 'idea' [*Vorstellung*] to cover all these occurrences, except decisions.

Now do thoughts belong to this inner world? Are they ideas? They are obviously not decisions. |

67 How are ideas distinct from the things of the outer world?

First: ideas cannot be seen, or touched, or smelled, or tasted, or heard.

I go for a walk with a companion. I see a green field, I thus have a visual impression of the green. I have it, but I do not see it.

Secondly: ideas are something we have. We have sensations, feelings, moods, inclinations, wishes. An idea that someone has belongs to the content of his consciousness.

The field and the frogs in it, the Sun which shines on them, are there no matter whether I look at them or not, but the sense impression I have of green exists only because of me, I am its owner. It seems absurd to us that a pain, a mood, a wish should go around the world without an owner, independently. A sensation is impossible without a sentient being. The inner world presupposes somebody whose inner world it is.

Thirdly: ideas need an owner. Things of the outer world are on the contrary independent.

My companion and I are convinced that we both see the same field) but each of us has a particular sense impression of green. I glimpse a strawberry among the green strawberry leaves. My companion cannot find it, he is colour-blind. The colour impression he gets from the strawberry is not noticeably different from the one he gets from the leaf.

should examine it from all sides. Something in itself not perceptible by sense, the thought is presented to the reader – and I must be content with that – wrapped up in a perceptible linguistic form. The pictorial aspect of language presents difficulties. The sensible always breaks in and makes expressions pictorial and so improper. So one fights against language, and I am compelled to occupy myself with language although it is not my proper concern here. I hope I have succeeded in making clear to my readers what I want to call 'thought'.

Now does my companion see the green leaf as red, or does he see the red berry as green, or does he see both with one colour which I am not acquainted with at all? These are unanswerable, indeed really nonsensical, questions. For when the word 'red' is meant not to state a property of things but to characterize sense impressions belonging to my consciousness, it is only applicable within the realm of my consciousness. For it is impossible to compare my sense impression with someone else's. For that, it would be necessary to bring together in one consciousness a sense impression belonging to one consciousness and a sense impression belonging to another consciousness. Now even if it were possible to make an idea disappear from one consciousness and at the same time make an idea appear in another consciousness, the question whether it is the same idea would still remain unanswerable. It is so much of the essence of any one of my ideas to be a content of my consciousness, that any idea someone else has is, just as such, different from mine. But might it not be possible that my ideas, the entire content of my consciousness, might be at the same time the content of a more embracing, perhaps divine consciousness? Only if I were myself part of the divine being. But then would they really be my ideas, would I be their owner? This so far oversteps the limits of human understanding that we must leave this possibility out of account. In any case it is impossible for us men to compare other people's ideas with our own. I pick the strawberry, I hold it between my fingers. Now my companion sees it too, this same strawberry; but each of us has his own idea. Nobody else has my idea, but many people can see the same thing. Nobody else has my pain. Someone may have sympathy with me, but still my pain belongs to me and his sympathy to him. He has not got my pain, and I have not got his feeling of sympathy.

Fourthly: every idea has only one owner; no two men have the same idea.

For otherwise it would exist independently of this man and independently of that man. Is that lime tree my idea? By using the expression 'that lime tree' in this question I am really already anticipating the answer, for I mean to use this expression to designate what I see and other people too can look at and touch. There are now two possibilities. If my intention is realized, if I do designate something with the expression 'that lime tree', then the thought expressed in the sentence 'That lime tree is my idea' must obviously be denied. But if my intention is not realized, if I only think I see without really seeing, if on that account the designation 'that lime tree' is empty, then I have wandered into the realm of fiction without knowing it or meaning to. In that case neither the content of the sentence 'That lime tree is my idea' nor the content of the sentence 'That lime tree is not my idea' is true, for in both cases I have a predication which lacks an object. So then I can

refuse to answer the question, on the ground that the content of the sentence 'That lime tree is my idea' is fictional. I have, of course, got an idea then, but that is not what I am using the words 'that lime tree' to designate. Now someone might really want to designate one of his ideas with the words 'that lime tree'. He would then be the owner of that which he wants to designate with those words, but then he would not see that lime tree and no one else would see it or be its owner.

I now return to the question: is a thought an idea? If other people can assent to the thought I express in the Pythagorean theorem just as I do, then it does not belong to the content of my consciousness, I am not its owner; yet I can, nevertheless, acknowledge it as true. However, if what is taken to be the content of the Pythagorean theorem by me and by somebody else is not the same thought at all, we should not really say '*the* Pythagorean theorem', but '*my* Pythagorean theorem', '*his* Pythagorean theorem', and these would be different, for the sense necessarily goes with the sentence. In that case my thought may be the content of my consciousness and his thought the content of his. Could the sense of my Pythagorean theorem be true and the sense of his false? I said that the word 'red' was applicable only in the sphere of my consciousness if it was not meant to state a property of things but to characterize some of my own sense impressions. Therefore the words 'true' and 'false', as I understand them, might also be applicable only in the realm of my consciousness, if they were not | meant to apply to something of which I was not the owner, but to characterize in some way the content of my consciousness. Truth would then be confined to this content and it would remain doubtful whether anything at all similar occurred in the consciousness of others.

If every thought requires an owner and belongs to the contents of his consciousness, then the thought has this owner alone; and there is no science common to many on which many could work, but perhaps I have my science, a totality of thoughts whose owner I am, and another person has his. Each of us is concerned with contents of his own consciousness. No contradiction between the two sciences would then be possible, and it would really be idle to dispute about truth; as idle, indeed almost as ludicrous, as for two people to dispute whether a hundred-mark note were genuine, where each meant [*meinte*] the one he himself had in his pocket and understood the word 'genuine' in his own particular sense. If someone takes thoughts to be ideas, what he then accepts as true is, on his own view, the content of his consciousness, and does not properly concern other people at all. If he heard from me the opinion that a thought is not an idea he could not dispute it, for, indeed, it would not now concern him.

So the result seems to be: thoughts are neither things in the external world nor ideas.

A third realm must be recognized. Anything belonging to this realm has it in common with ideas that it cannot be perceived by the senses, but has it in common with things that it does not need an owner so as to belong to the contents of his consciousness. Thus for example the thought we have expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no owner. It is not true only from the time when it is discovered; just as a planet, even before anyone saw it, was in interaction with other planets.<sup>E</sup>

But I think I hear an odd objection. I have assumed several times that the same thing as I see can also be observed by other people. But what if everything were only a dream? If I only dreamed I was walking in the company of somebody else, if I only dreamed that my companion saw the green field as I did, if it were all only a play performed on the stage of my consciousness, it would be doubtful whether there were things of the external world at all. Perhaps the realm of things is empty and I do not see any things or any men, but only have ideas of which I myself am the owner. An idea, being something which can no more exist independently of me than my feeling of fatigue, cannot be a man, cannot look at the same field together with me, cannot see the strawberry I am holding. It is quite incredible that I really have only my inner world, instead of the whole environment in which I supposed myself to move and to act. And yet this is an inevitable consequence of the thesis that only what is my idea can be the object of my awareness. What would follow from this thesis if it were true? Would there then be other men? It would be possible, but I should know nothing of them. For a man cannot be my idea; consequently, if our thesis were true, he cannot be an object of my awareness either. And so this would undercut any reflections in which I assumed that something was an object for somebody else as it was for myself, for even if this were to happen I should know nothing of it. It would be impossible for me to distinguish something owned by myself from something I did not own. In judging something not to be my idea I would make it into the object of my thinking and, therefore, into my idea. On this view, is there a green field? Perhaps, but it would not be visible to me. For if a field is not my idea, it cannot, according to our thesis, be an object of my awareness. But if it is my idea it is invisible, for ideas are not visible. I can indeed have the idea of a green field; but this is not green, for there are no green ideas. Does a missile weighing a hundred kilogrammes exist, according to this view? Perhaps, but I could know nothing of it.

<sup>E</sup> A person sees a thing, has an idea, grasps or thinks a thought. When he grasps or thinks a thought he does not create it but only comes to stand in a certain relation to what already existed — a different relation from seeing a thing or having an idea.

If a missile is not my idea, then, according to our thesis, it cannot be an object of my awareness, of my thinking. But if a missile were my idea, it would have no weight. I can have an idea of a heavy missile. This then contains the idea of weight as a constituent idea. But this constituent idea is not a property of the whole idea, any more than Germany is a property of Europe. So the consequence is:

Either the thesis that only what is my idea can be the object of my awareness is false, or all my knowledge and perception is limited to the range of my ideas, to the stage of my consciousness. In this case I should have only an inner world and I should know nothing of other people.

It is strange how, in the course of such reflections, opposites turn topsy-turvy. There is, let us suppose, a physiologist of the senses. As is proper for someone investigating nature scientifically, he is at the outset far from supposing the things that he is convinced he sees and touches to be his own ideas. On the contrary, he believes that in sense impressions he has the most reliable evidence of things wholly independent of his feeling, imagining, thinking, which have no need of his consciousness. So little does he consider nerve fibres and ganglion cells to be the content of his consciousness that he is on the contrary inclined to regard his consciousness as dependent on nerve fibres and ganglion cells. He establishes that light rays, refracted in the eye, strike the visual nerve endings and there bring about a change, a stimulus. From this something is transmitted through nerve fibres to ganglion cells. Further processes in the nervous system perhaps follow upon this, and colour impressions arise, and these perhaps combine to make up what we call the idea of a tree. Physical, chemical and physiological occurrences get in between the tree and my idea. Only occurrences in my nervous system are immediately connected with my consciousness – or so it seems – and every observer of the tree has his particular occurrences in his particular nervous system. Now light rays, before they enter my eye, may be reflected by a mirror and diverge as if they came from places behind the mirror. The effects on the visual nerves and all that follows will now take place just as they would if the light rays had come from a tree behind the mirror and had been propagated undisturbed to the eye. So an idea of a tree will finally occur even though such a tree does not exist at all. The refraction of light too, with the mediation of the eye and nervous system, may give rise to an idea to which nothing at all corresponds. But the stimulation of the visual nerves need not even happen because of light. If lightning strikes near us, we believe we see flames, even though we cannot see the lightning itself. In this case the visual nerve is perhaps stimulated by electric currents occurring in our body as a result of the flash of lightning. If the visual nerve is stimulated by this means in just the way it would be stimulated by light rays coming from

flames, then we believe we see flames. It just depends on the stimulation of the visual nerve, no matter how that itself comes about.

We can go a step further. Properly speaking this stimulation of the visual nerve is not immediately given; it is only an hypothesis. We believe that a thing independent of us stimulates a nerve and by this means produces a sense impression; but strictly speaking we experience only that end of the process which impinges on our consciousness. Might not this sense impression, this sensation, which we attribute to a nerve stimulation, have other causes also, just as the same nerve stimulation may arise in different ways? If we call what happens in our consciousness an idea, then we really experience only ideas, not their causes. And if the scientist wants to avoid all mere hypothesis, then he is left just with ideas; everything dissolves into ideas, even the light rays, nerve fibres and ganglion cells from which he started. So he finally undermines the foundations of his own construction. Is everything an idea? Does everything need an owner without which it could have no existence? I have considered myself as the owner of my ideas, but am I not myself an idea? It seems to me as if I were lying in a deck-chair, as if I could see the toes of a pair of polished boots, the front part of a pair of trousers, a waistcoat, buttons, parts of a jacket, in particular the sleeves, two hands, some hair of a beard, the blurred outline of a nose. Am I myself this entire complex of visual impressions, this aggregate idea? It also seems to me as if I saw a chair over there. That is an idea. I am not actually much different from the chair myself, | for am I not myself just a complex of sense impressions, an idea? But where then is the owner of these ideas? How do I come to pick out one of these ideas and set it up as the owner of the rest? Why need this chosen idea be the idea I like to call 'I'? Could I not just as well choose the one that I am tempted to call a chair? Why, after all, have an owner for ideas at all? An owner would anyhow be something essentially different from ideas that were just owned; something independent, not needing any extraneous owner. If everything is idea, then there is no owner of ideas. And so now once again I experience opposites turning topsyturvy. If there is no owner of ideas then there are also no ideas, for ideas need an owner and without one they cannot exist. If there is no ruler, there are also no subjects. The dependence which I found myself induced to ascribe to the sensation, as contrasted with the sentient being, disappears if there no longer is any owner. What I called ideas are then independent objects. No reason remains for granting an exceptional position to that object which I call 'I'.

But is that possible? Can there be an experience without someone to experience it? What would this whole play be without a spectator? Can there be a pain without someone who has it? Being felt necessarily goes with pain, and furthermore someone feeling it necessarily goes with its



being felt. But then there is something which is not my idea and yet can be the object of my awareness, of my thinking; I myself am such a thing. Or can I be one part of the content of my consciousness, while another part is, perhaps, an idea of the Moon? Does this perhaps take place when I judge that I am looking at *the Moon*? Then this first part would have a consciousness, and part of the content of this consciousness would be I myself once more. And so on. Yet it is surely inconceivable that I should be inside myself like this in an infinite nest of boxes, for then there would not be just one I but infinitely many. I am not my own idea; and when I assert something about myself, e.g. that I am not feeling any pain at the moment, then my judgement concerns something which is not a content of my consciousness, is not an idea, namely myself. Therefore that about which I state something is not necessarily my idea. But someone perhaps objects: if I think I have no pain at the moment, does not the word 'I' answer to something in the content of my consciousness? And is that not an idea? That may be so. A certain idea in my consciousness may be associated with the idea of the word 'I'. But then this is one idea among other ideas, and I am its owner as I am the owner of the other ideas. I have an idea of myself, but I am not identical with this idea. What is a content of my consciousness, my idea, should be sharply distinguished from what is an object of my thinking. Therefore the thesis that only what belongs to the content of my consciousness can be the object of my awareness, of my thinking, is false. |

73 Now the way is clear for me to acknowledge another man likewise as an independent owner of ideas. I have an idea of him, but I do not confuse it with him himself. And if I state something about my brother, I do not state it about the idea that I have of my brother.

The patient who has a pain is the owner of this pain, but the doctor who is treating him and reflects on the cause of this pain is not the owner of the pain. He does not imagine he can relieve the pain by anaesthetizing himself. There may very well be an idea in the doctor's mind that answers to the patient's pain, but that is not the pain, and is not what the doctor is trying to remove. The doctor might consult another doctor. Then one must distinguish: first, the pain, whose owner is the patient; secondly, the first doctor's idea of this pain; thirdly, the second doctor's idea of this pain. This last idea does indeed belong to the content of the second doctor's consciousness, but it is not the object of his reflection; it is rather an aid to reflection, as a drawing may be. The two doctors have as their common object [of thinking] the patient's pain, which they do not own. It may be seen from this that not only a thing but also an idea may be a common object of thinking for people who do not have the idea.

In this way, it seems to me, the matter becomes intelligible. If man

could not think and could not take as the object of his thinking something of which he was not the owner, he would have an inner world but no environment. But may this not be based on a mistake? I am convinced that the idea I associate with the words 'my brother' corresponds to something that is not my idea and about which I can say something. But may I not be making a mistake about this? Such mistakes do happen. We then, against our will, lapse into fiction. Yes, indeed! By the step with which I win an environment for myself I expose myself to the risk of error. And here I come up against a further difference between my inner world and the external world. I cannot doubt that I have a visual impression of green, but it is not so certain that I see a lime leaf. So, contrary to widespread views, we find certainty in the inner world, while doubt never altogether leaves us in our excursions into the external world. But the probability is nevertheless in many cases hard to distinguish from certainty, so we can venture to judge about things in the external world. And we must make this venture even at the risk of error if we do not want to fall into far greater dangers.

As the result of these last considerations I lay down the following: not everything that can be the object of my acquaintance is an idea. I, as owner of ideas, am not myself an idea. Nothing now stops me from acknowledging other men to be owners of ideas, just as I am myself. And, once given the possibility, the probability is very great, so great that it is in my opinion no longer distinguishable from certainty. Would there be a science of history otherwise? Would not all moral theory, all law, otherwise collapse? What would be left of religion? The natural sciences too could only be assessed as fables like astrology and alchemy. Thus the reflections I have set forth on the assumption that there are other men besides myself, who can make the same thing the object of their consideration, their thinking, remain in force without any essential weakening.

Not everything is an idea. Thus I can also acknowledge thoughts as independent of me; other men can grasp them just as much as I; I can acknowledge a science in which many can be engaged in research. We are not owners of thoughts as we are owners of our ideas. We do not *have* a thought as we have, say, a sense impression, but we also do not *see* a thought as we see, say, a star. So it is advisable to choose a special expression; the word 'grasp' suggests itself for the purpose.<sup>F</sup> To the grasping of thoughts there must then correspond a special mental capacity, the power of thinking. In thinking we do not produce thoughts, we

<sup>F</sup> The expression 'grasp' is as metaphorical as 'content of consciousness'. The nature of language does not permit anything else. What I hold in my hand can certainly be regarded as the content of my hand; but all the same it is the content of my hand in quite another and a more extraneous way than are the bones and muscles of which the hand consists or again the tensions these undergo.

grasp them. For what I have called thoughts stand in the closest connection with truth. What I acknowledge as true, I judge to be true quite apart from my acknowledging its truth or even thinking about it. That someone thinks it has nothing to do with the truth of a thought. 'Facts, facts, facts' cries the scientist if he wants to bring home the necessity of a firm foundation for science. What is a fact? A fact is a thought that is true. But the scientist will surely not acknowledge something to be the firm foundation of science if it depends on men's varying states of consciousness. The work of science does not consist in creation, but in the discovery of true thoughts. The astronomer can apply a mathematical truth in the investigation of long past events which took place when – on Earth at least – no one had yet recognized that truth. He can do this because the truth of a thought is timeless. Therefore that truth cannot have come to be only upon its discovery.

Not everything is an idea. Otherwise psychology would contain all the sciences within it, or at least it would be the supreme judge over all the sciences. Otherwise psychology would rule even over logic and mathematics. But nothing would be a greater misunderstanding of mathematics than making it subordinate to psychology. Neither logic nor mathematics has the task of investigating minds and contents of consciousness owned by individual men. Their task could perhaps be represented rather as the investigation of *the* mind; of *the* mind, not of minds. |

75 The grasp of a thought presupposes someone who grasps it, who thinks. He is the owner of the thinking, not of the thought. Although the thought does not belong with the contents of the thinker's consciousness, there must be something in his consciousness that is aimed at the thought. But this should not be confused with the thought itself. Similarly Algol itself is different from the idea someone has of Algol.

A thought belongs neither to my inner world as an idea, nor yet to the external world, the world of things perceptible by the senses.

This consequence, however cogently it may follow from the exposition, will nevertheless perhaps not be accepted without opposition. It will, I think, seem impossible to some people to obtain information about something not belonging to the inner world except by sense perception. Sense perception indeed is often thought to be the most certain, even the sole, source of knowledge about everything that does not belong to the inner world. But with what right? For sense perception has as necessary constituents our sense impressions and these are a part of the inner world. In any case two men do not have the same sense impressions though they may have similar ones. Sense impressions alone do not reveal the external world to us. Perhaps there is a being that has only sense impressions without seeing or touching things. To have visual impressions is not to see things. How does it happen that I see the tree just there where I do see it? Obviously it depends on the visual impres-

sions I have and on the particular sort which occur because I see with two eyes. On each of the two retinas there arises, physically speaking, a particular image. Someone else sees the tree in the same place. He also has two retinal images but they differ from mine. We must assume that these retinal images determine our impressions. Consequently the visual impressions we have are not only not the same, but markedly different from each other. And yet we move about in the same external world. Having visual impressions is certainly necessary for seeing things, but not sufficient. What must still be added is not anything sensible. And yet this is just what opens up the external world for us; for without this non-sensible something everyone would remain shut up in his inner world. So perhaps, since the decisive factor lies in the non-sensible, something non-sensible, even without the co-operation of sense impressions, could also lead us out of the inner world and enable us to grasp thoughts. Outside our inner world we should have to distinguish the external world proper of sensible, perceptible things and the realm of what is non-sensibly perceptible. We should need something non-sensible for the recognition of both realms; but for the sense perception of things we should need sense impressions as well, and these belong entirely to the inner world. So the distinction between the ways in which a thing and a thought are given mainly consists in something which is assignable, not to either of the two realms, but to the inner world. Thus I cannot find this distinction to be so great as to make impossible the presentation of a thought that does not belong to the inner world. |

70 A thought, admittedly, is not the sort of thing to which it is usual to apply the term 'actual' [*wirklich*]. The world of actuality is a world in which this acts [*wirkt*] on that and changes it and again undergoes reactions [*Gegenwirkungen*] itself and is changed by them. All this is a process in time. We will hardly admit what is timeless and unchangeable to be actual. Now is a thought changeable or is it timeless? The thought we express by the Pythagorean theorem is surely timeless, eternal, unvarying. But are there not thoughts which are true today but false in six months' time? The thought, for example, that the tree there is covered with green leaves, will surely be false in six months' time. No, for it is not the same thought at all. The words 'This tree is covered with green leaves' are not sufficient by themselves to constitute the expression of thought, for the time of utterance is involved as well. Without the time-specification thus given we have not a complete thought, i.e. we have no thought at all. Only a sentence with the time-specification filled out, a sentence complete in every respect, expresses a thought. But this thought, if it is true, is true not only today or tomorrow but timelessly. Thus the present tense in 'is true' does not refer to [*deutet . . . auf*] the speaker's present; it is, if the expression be permitted, a tense of timelessness. If we merely use the assertoric sentence-form and avoid the word

'true', two things must be distinguished, the expression of the thought and assertion. The time-specification that may be contained in the sentence belongs only to the expression of the thought; the truth, which we acknowledge by using the assertoric sentence-form, is timeless. To be sure the same words, on account of the variability of language with time, may take on another sense, express another thought; this change, however, relates only to the linguistic realm.

And yet what value could there be for us in the eternally unchangeable, which could neither be acted upon nor act on us? Something entirely and in every respect inactive would be quite unactual, and so far as we are concerned it would not be there. Even the timeless, if it is to be anything for us, must somehow be implicated with the temporal. What would a thought be for me if it were never grasped by me? But by grasping a thought I come into a relation to it, and it to me. It is possible that the same thought as is thought by me today was not thought by me yesterday. Of course this does away with strict timelessness. But we may be inclined to distinguish between essential and inessential properties and to regard something as timeless if the changes it undergoes involve only inessential properties. A property of a thought will be called inessential if it consists in, or follows from, the fact that this thought is grasped by a thinker.

77 How does a thought act? By being grasped and taken to be true. This is a process in the inner world of a thinker which may have further consequences in this inner world, and which may also encroach on the sphere of the will and make itself noticeable in the outer world as well. If, for example, I grasp the thought we express by the theorem of Pythagoras, the consequence may be that I recognize it to be true, and further that I apply it in making a decision, which brings about the acceleration of masses. This is how our actions are usually led up to by acts of thinking and judging. And so thoughts may indirectly influence the motion of masses. The influence of man on man is brought about for the most part by thoughts. People communicate thoughts. How do they do this? They bring about changes in the common external world, and these are meant to be perceived by someone else, and so give him a chance to grasp a thought and take it to be true. Could the great events of world history have come about without the communication of thoughts? And yet we are inclined to regard thoughts as unactual, because they appear to do nothing in relation to events, whereas thinking, judging, stating, understanding, in general doing things, are affairs that concern men. How very different the actuality of a hammer appears, compared with that of a thought! How different a process handing over a hammer is from communicating a thought! The hammer passes from one control to another, it is gripped, it undergoes pressure, and thus its density, the disposition of its parts, is locally changed. There is nothing

of all this with a thought. It does not leave the control of the communicator by being communicated, for after all man has no power over it. When a thought is grasped, it at first only brings about changes in the inner world of the one who grasps it; yet it remains untouched in the core of its essence, for the changes it undergoes affect only inessential properties. There is lacking here something we observe everywhere in physical process – reciprocal action. Thoughts are not wholly unactual but their actuality is quite different from the actuality of things. And their action is brought about by a performance of the thinker; without this they would be inactive, at least as far as we can see. And yet the thinker does not create them but must take them as they are. They can be true without being grasped by a thinker; and they are not wholly unactual even then, at least if they *could* be grasped and so brought into action.