FC disjunction in state-based semantics

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1 Introduction

In a state-based semantics formulas are interpreted with respect to states rather than possible worlds. States are less determinate entities than worlds and can be identified with truthmakers (van Fraassen, 1969; Fine, 2017), possibilities (Humberstone, 1981; Holliday, 2015), situations (Barwise and Perry, 1983), information states (Veltman, 1985, 1996; Dekker, 2012) and more.[1] The partial nature of a state makes a state-based semantics particularly suitable to capture various aspects of disjunctive words in natural language, including their indeterminate, epistemic and choice-offering nature.

There are at least three ways to define disjunction in a state-based semantics: The first notion, $\vee_1$, is from possibility semantics and dynamic semantics (Humberstone, 1981; Heim, 1983; Groenendijk and Stokhof, 1991); the second notion $\vee_2$ has been independently proposed in team logic (Yang and Väänänen, 2017) and assertability logic (Hawke and Steinert-Threlkeld, 2016) (but see also Cresswell, 2004); the third notion $\vee_3$ is used in inquisitive semantics (Ciardelli and Roelofsen, 2011) and in some versions of truthmaker semantics (van Fraassen, 1969; Fine, 2017).

In the first part of this draft, I will compare these notions with emphasis on their potential to account for narrow and wide scope Free Choice (FC) inferences when combined with a possibility modal. While assertability logic $\vee_2$ in combination with a context-sensitive notion of modality derives *wide* scope FC inference (Hawke and Steinert-Threlkeld, 2016) and inquisitive/truthmaker $\vee_3$ combined with alternative-sensitive notions of modality derive *narrow* scope

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[1] As in a Kripke semantics for intuitionistic logic, states are often represented as points in a partially ordered set. In this material, with admittedly some loss of generality (Fine, 2017), I will identify states with sets of possible worlds. The powerset of any set is isomorphic to a complete atomic Boolean lattice. One of the advantages of defining states as elements of a partially ordered set rather than as a set (of worlds) is that one may want to include partially ordered sets not satisfying the properties of Boolean lattices. There are two main reasons for my choice to characterise states as sets of worlds nevertheless: (i) I will mainly focus on linguistic applications and I have not yet found any compelling linguistic evidence favouring the more abstract algebraic characterisation; (ii) this material deals with disjunction and the notion of disjunction I will eventually adopt is more perspicuously defined if we characterise states as sets rather than points.

FC inference (Aloni, 2007b; Aloni and Ciardelli, 2013),[2] none of the existing combinations accounts for both wide and narrow scope FC. Furthermore, when FC inducing sentences occur under negation, these systems predict weaker readings than attested in ordinary language use. In the second part, I will present a new state-based system, which, adopting an enriched version of $\vee_2$, derives both wide and narrow scope FC while solving the negation problem.

2 The paradox of free choice

Sentences of the form "You may A or B" are normally understood as implying "You may A and you may B". The following, however, is not a valid principle in classical deontic logic (von Wright, 1968).

(1) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice Principle]

As Kamp (1973) pointed out, plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do because it would allow us to derive any $\Diamond q$ from $\Diamond p$ as shown in (2):

(2)
1. $\Diamond p$ [assumption]
2. $\Diamond(p \vee q)$ [from 1, by principle (3)]
3. $\Diamond q$ [from 2, by free choice principle]

The step leading to 2 in the derivation above uses the following valid principle of classical modal logic:

(3) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$

In natural language, however, (3) seems invalid (*You may go to the beach* doesn't seem to imply *You may go to the beach or the cinema*), while (1) seems to hold, in direct opposition to the principles of deontic logic. Von Wright (1968) called this the paradox of free choice permission. Related paradoxes arise also for imperatives (see Ross' paradox), and other modal constructions.

Several solutions have been proposed to the paradox of free choice. Many have argued that what we called the Free Choice Principle is merely a pragmatic inference and therefore the step leading to 3 in derivation (2) is unjustified. Various ways of deriving free choice inferences as conversational implicatures have been proposed (e.g., Gazdar 1979, Kratzer and Shimoyama 2002, Schulz 2005, Fox, 2007 and Franke, 2011). One argument in favour of such a pragmatic account comes from the observation that free choice effects disappear in negative contexts. For example, sentence (4) cannot merely mean that no one is allowed to eat the cake and the ice-cream (reading (4-b)), as we would expect if free choice effects were semantic entailments rather than pragmatic implicatures (Alonso-Ovalle, 2006):

(4) No one is allowed to eat the cake or the ice-cream.
a. $\equiv \neg\exists x\Diamond(\phi(x) \vee \psi(x))$
b. $\not\equiv \neg\exists x(\Diamond\phi(x) \wedge \Diamond\psi(x))$

[2] See also Fine (2017) who deals with choice offering imperatives employing an 'alternative-sensitive' notion of logical consequence.

Others have proposed modal logic systems where the step leading to 3 in (2) is justified but the step leading to 2 is no longer valid, e.g., Aloni (2007), who proposes a uniform semantic account of free choice effects of disjunctions and indefinites under both modals and imperatives.[3]

The system I will present later in this paper is of this latter kind and indeed in such a system only a restricted version of principle (3) will be valid. Furthermore, contrary to most analyses mentioned so far this system will not only derive narrow scope free choice inferences, i.e. inference where disjunction takes narrow scope with respect to the modal operator, but also wide scope examples of free choice:

(5) FC inferences

a. Wide scope FC: $\Diamond a \vee \Diamond b \rightsquigarrow \Diamond a \wedge \Diamond b$
b. Narrow scope FC: $\Diamond(a \vee b) \rightsquigarrow \Diamond a \wedge \Diamond b$

Here are two classical linguistic examples illustrating free choice inferences with deontic and epistemic modals of the narrow and wide scope kind:

(6) Deontic FC [Kamp 1973]

a. You may go to the beach or (you may go) to the cinema.
b. $\rightsquigarrow$ You may go to the beach and you may go to the cinema.

(7) Epistemic FC [Zimmermann 2000]

a. Mr. X might be in Victoria or (he might be) in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria and he might be in Brixton.

Wide scope FC leads to similar paradoxes as narrow scope FC, as illustrated in (9) involving plain addition ($\alpha \to \alpha \vee \beta$) rather than the modal principle in (3) and a wide scope version of the Free Choice Principle:

(8) $\Diamond\alpha \vee \Diamond\beta \to \Diamond\alpha$ [Wide Scope Free Choice Principle]

(9)
1. $\Diamond p$ [assumption]
2. $\Diamond p \vee \Diamond q$ [from 1, by addition]
3. $\Diamond q$ [from 2, by wide scope free choice principle]

Again one could argue that principle (8) should be pragmatically derived[4] or, if derived semantically, as we will do, addition will have to be restricted.

At this point we should also mention an alternative account to wide scope free choice inference given by Zimmermann (2000) and further refined by Geurts (2005). Zimmermann (2000) proposes a modal analysis of linguistic disjunction, which, as (10) illustrates, should be treated as a conjunctive list of epistemic possibilities:

(10) A or $B \mapsto \Diamond\alpha \wedge \Diamond\beta$, where $\Diamond$ is an epistemic possibility operator

[3]Simons (2005) and Barker (2011) also proposed semantic accounts of free choice inferences, the latter crucially employing an analysis of *or* in terms of linear logic additive disjunction combined with a representation of strong permission using the deontic reduction strategy as in Lokhorst (2006).

[4]It is not trivial to derive (8) by Gricean means. Most pragmatic analyses of FC inference indeed only derive the narrow scope case (one exception is Schulz, 2005) and attempt to reduce all surface wide scope free choice examples to cases of narrow scope free choice. One argument against this reductive strategy will be given below.

Zimmermann then distinguishes between (8), which, according to him, is an unjustified logical principle, and the following intuitively valid natural language principle:

(11) X may A or may B $\rightsquigarrow$ X may A and X may B

By analysing disjunctions as conjunctions of epistemic possibilities, as in (10), Zimmermann argues that the correct logical rendering of (11) is (12), which, if derived, explains our wide scope FC intuitions:[5]

(12) $(\Diamond P\alpha \wedge \Diamond P\beta) \rightarrow (P\alpha \wedge P\beta)$ where $\Diamond$ is an epistemic possibility operator and P a deontic possibility operator

The system I will present in section 4 shares the basic intuition of Zimmermann's analysis, namely that when one says *A* or *B*, one normally conveys that each disjunct is an open option. My implementation of this idea, however, will be rather different from Zimmermann's or Geurts'.

I would like to conclude this section with some remarks on the question whether free choice inference is semantics or pragmatics. One of the reasons why this question is still open is that arguments for and against semantic or pragmatic approaches to free choice phenomena are often inconclusive. For example, the observation that free choice effects disappear in negative contexts (see discussion around example (4) above), which is normally taken to favour pragmatic approaches, can be accounted for in semantic approaches like Willer (2018) or the one I will present below, while any 'pragmatic' system which predicts the availability of embedded implicatures (Chierchia *et al.*, 2011) needs adjustments to account for these facts.

On my view, FC inferences are neither purely semantic nor purely pragmatic, rather they are *inferences of the third kind*:[6] they are typically derivable by conversational principles (at least the narrow scope variant) but lack other defining properties of pragmatic inference: they are often non-cancellable, they are sometimes embeddable[7] and recent experiments have shown that their processing time equals that of literal interpretations, being much easier to process than, for example, scalar implicatures (Chemla and Bott, 2014). My hypothesis is that there are at least two sorts of "inferences of the third kind": (a) those resulting from historical processes of conventionalisation of originally pragmatic inferences (*fossilised implicatures*, typically embeddable) (Aloni, 2012; Aloni and Franke, 2012) and (b) those that follow from automatised reasoning about the current conversational situation (*automatised implicatures*, typically non-

[5] Zimmermann actually only derives the weaker principle in (i) (under certain assumptions including his Authority principle). $\Box\alpha$ should be read here as "it is certain that α":

(i) $(\Diamond P\alpha \wedge \Diamond P\beta) \rightarrow (\Box P\alpha \wedge \Box P\beta)$

Geurts' (2005) refinement avoids this problem.

[6] Maybe a more appropriate (even though less catchy) name would have been "inference of the fourth or fifth kind" because more kinds of inference at the semantics and pragmatic interface have been discussed in the literature including presupposition and conventional implicature. The negation facts illustrated by example (4) prevent treating FC inferences as conventional implicatures or presupposition since these normally project under negation.

[7] As in the case of FC indefinites like Spanish *cualquiera* (Menéndez-Benito, 2010) or Italian *qualunque* (Chierchia, 2013; Aloni, 2007a) or so called universal FC cases experimentally investigated in Chemla (2009).

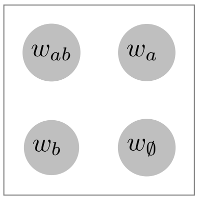


Figure 1: Logical space for $A = \{a, b\}$

embeddable) (the present paper). The goal is to arrive at predictive models of these phenomena, including models of (a) the historical processes of conventionalisation of pragmatic inferences and (b) the automatisation of reasoning about the current conversational situation. The overarching goal is to eventually develop an alternative architecture, replacing the traditional divide between truth-conditional semantics and Gricean pragmatics, where all these inferences find their natural place. The system I will present in section 4 is a first step towards this objective.

3 State-based semantics: three disjunctions

We start with the language of classical propositional modal logic.

Definition 1 (Language)

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Diamond\phi$$

where $p \in A$.

A model for such a language is a triple $M = \langle W, R, V\rangle$ as in standard Kripke semantics, where W is a set of worlds, R is an accessibility relation and V is a world dependent valuation function for the atoms in A. Contrary to the classical case, however, formulas here will be interpreted with respect to sets of possible worlds rather than single worlds. We call these sets states. A state s in a given model $M = \langle W, R, V\rangle$ is a subset of W.

We could identify possible worlds with valuation functions and then the logical space for a language containing only two sentential atoms a and b would consist of 4 worlds, which can be represented as in Figure 1, where w_a stands for a world where only a is true, w_b only b, etc. This set of possible worlds will be used for illustration throughout the paper.

We define the notion of support (or truth) for an arbitrary formula in L at a state s in a model M (explicit reference to the model will be suppressed later on). We start with the atomic clause and the clauses for conjunction and negation.

Definition 2 (Basic semantic clauses)

$$\begin{array}{rcl} M, s \models p & \textit{iff} & \forall w \in s : V(w, p) = 1 \\ M, s \models \phi \wedge \psi & \textit{iff} & M, s \models \phi \ \&\ M, s \models \psi \\ M, s \models \neg\phi & \textit{iff} & \forall w \in s : M, \{w\} \not\models \phi \end{array}$$

An atomic formula is supported at a state s only if true in all worlds in s. A conjunction is supported at s just when each conjunct is supported at s. As for negation, for the moment we adopt an intuitionistic-like notion: a negation is supported in s just in case the formula negated is not supported at any of the singleton substates of s. Logical consequence is defined in terms of preservation of support.

Definition 3 (Logical consequence) $\phi \models \psi$ *iff* $\forall M, s : M, s \models \phi \;\Rightarrow\; M, s \models \psi$

A property of formulas which will be useful for our comparison later on is distributivity. A formula ϕ is distributive if in any model M, ϕ is supported by a state s iff ϕ is supported by all singleton substates of s.

Definition 4 (Distributivity) *ϕ is distributive, if*

$$\forall M, s : M, s \models \phi \;\Leftrightarrow\; \forall w \in s : M, \{w\} \models \phi$$

The left to right direction in the condition for distributivity relates to the notion of persistence (or monotonicity) in intuitionistic logic, possibility semantics and data semantics.

Here are some consequences of these definitions.

Fact 1 *p, $\neg\phi$ are distributive;*

Fact 2 *$\emptyset \models \phi$, if ϕ is distributive.*

So far the consequence relation is classical (as in Humberstone 1981); but bivalence fails, e.g. for $s = \{w_a, w_b\}$, $s \not\models a \;\;\&\;\; s \not\models \neg a$.

This baseline semantics can be extended with at least three different notions of disjunction. These three notions collapse if s is a singleton, which corresponds to the classical case:

Definition 5 (Three notions of disjunction)

$$\begin{array}{lll} s \models \phi \vee_1 \psi & \textit{iff} & \forall w \in s : \{w\} \models \phi \textit{ or } \{w\} \models \psi \\ s \models \phi \vee_2 \psi & \textit{iff} & \exists t, t' : t \cup t' = s \;\&\; t \models \phi \;\&\; t' \models \psi \\ s \models \phi \vee_3 \psi & \textit{iff} & s \models \phi \textit{ or } s \models \psi \end{array}$$

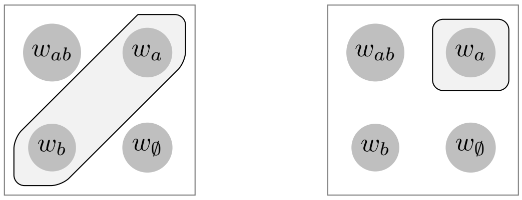
The first notion, $\vee_1$, is equivalent to the notion of disjunction adopted in possibility semantics and standard dynamic semantics (e.g. DPL). It is what one gets if one defines disjunction in terms of the given notion of negation and conjunction:

Fact 3 $\phi \vee_1 \psi \equiv \neg(\neg\phi \wedge \neg\psi)$

But note that if we had adopted a weaker/classical-style notion of negation ($s \models \neg\phi$ iff $s \not\models \phi$), then the equation $\phi \vee \psi = \neg(\neg\phi \wedge \neg\psi)$ would have generated $\vee_3$.

According to the second notion, $\vee_2$, from team and assertability logic, a disjunction is supported by s if s is the union of two states each supporting one of the disjuncts. A type 3 disjunction, $\vee_3$, from inquisitive semantics, instead is supported in s iff s supports at least one of the disjuncts.

Disjunction 3 leads to a failure of the law of excluded middle while disjunction 1 and 2 are classical in this respect:



(a) $\models (a \vee_{1/2/*3} b)$ (b) $\models (a \vee_{1/2/3} b)$

Figure 2: Comparison $\vee_1, \vee_2$ and $\vee_3$

Fact 4 $\models \phi \vee_{1/2} \neg\phi$, *but* $\not\models \phi \vee_3 \neg\phi$

For distributive sentences the first two notions are equivalent, the third notion instead is strictly stronger: whenever a disjunction 3 is supported also the other disjunctions are supported, but not the other way around:

Fact 5 *If* ϕ, ψ *are distributive,*

(i) $\phi \vee_1 \psi \equiv \phi \vee_2 \psi$

(ii) $\phi \vee_3 \psi \models \phi \vee_{1/2} \psi$

(iii) $\phi \vee_{1/2} \psi \not\models \phi \vee_3 \psi$

Clause (ii) in fact 5 is trivial for disjunction 1. For disjunction 2 it depends on the fact that one of the two substates necessary for verifying $\phi \vee_2 \psi$ can be empty, so whenever the whole state supports one of the disjuncts (see state (b) in Figure 2), we can find two suitable substates supporting each disjunct, the state itself and the empty state.

State (a) in Figure 2 shows that we can find a state which supports disjunctions 1 and 2 but not disjunction 3. This counterexample is very instructive because it illustrates the different conceptualisations which lie behind these three notions: $\vee_1$ and $\vee_2$ make sense if we read the support relation as modelling the assertability of a sentence in an information state:

- $s \models \phi$ means "agent in state s has enough evidence to assert ϕ"

$\vee_3$ instead presupposes a different interpretation for the notion of support as for example in inquisitive semantics or truthmaker semantics where $s \models \phi$ more naturally reads as:

- "s contains enough information to resolve (the issues raised by) ϕ" (inquisitive semantics)
- "ϕ is true because of fact s" (truthmaker semantics)

In inquisitive semantics, $\models$ models resolution of a sentence (either interrogative or declarative) rather than its assertability. In the inquisitive framework, a disjunction is taken to raise an issue, the issue which of the disjuncts is true. It is clear that state (a) in Figure 2 does not contain enough information to resolve this issue while state (b) does. In truthmaker semantics, states stand more for pieces of the world (facts, situations) than for pieces of information and "disjunctive facts" (as would be the one represented by state (a) in Figure 2) are normally excluded. State (b) instead is a typical example of a truthmaker for the given disjunctive sentence.

The counterexample illustrated by state (a) further shows that the adoption of $\vee_1$ and $\vee_2$ allows a direct account of the indeterminacy of natural language disjunction illustrated, for example, by example (13) from Grice (1991, p. 82):

(13) A: X or Y will be elected.
B: That's not so; X or Y or Z will be elected.

B's reply in (13) demonstrates that disjunctions can be taken to be true without either disjunct's being true. A system adopting $\vee_3$ would need extra machinery to account for these cases (e.g. the addition of a silent modal operator, as in Veltman's Data Semantics, or a shift to set of states, as in Inquisitive Semantics).

Finally our three "disjunctions" further differ in the notion of semantic content they allow to define. In standard logic-based analyses of linguistic meanings, the semantic content of a sentence ϕ is typically defined as the set of evaluation points which verify ϕ. Normally, evaluation points are identified with possible worlds and so the semantic content of a sentence is identified with a set of possible worlds (a proposition), those worlds where the sentence is true. In a state-based semantics, semantic content can be defined as the set of *states* supporting the sentence, so in my characterisation, a set of sets of possible worlds.

Definition 6 (Semantic content) $[\phi]_M = \{s \subseteq W \mid s \models \phi\}$

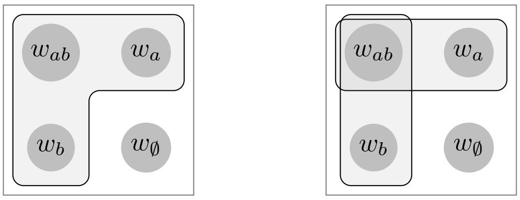
It is easy to see that adopting $\vee_3$ gives rise to a so called inquisitive semantic content for disjunctive sentences, i.e., a semantic content containing more than one maximal state (content (b) in Figure 3); while disjunctions 1 and 2 give rise to a non-inquisitive, classical notion of semantic content (content (a) in Figure 3).

Fact 6 *Let ϕ, ψ be distributive and logically independent.*

1. $\{s \mid s \models (\phi \vee_3 \psi)\}$ *is inquisitive, i.e. it contains more than one maximal state, aka alternative;*
2. $\{s \mid s \models (\phi \vee_{1/2} \psi)\}$ *is not inquisitive.*

This feature of inquisitive $\vee_3$ has been defended because it models the alternative-inducing nature of disjunctive words in natural language. The main function of a disjunction on this view is to present a set of alternatives, uttering *A* or *B* corresponds to introducing two options for consideration: the alternative that *A* is the case and the alternative that *B* is the case. This alternative-inducing nature of $\vee_3$ has been recently employed in formal semantics to capture various linguistic phenomena including the simplification of disjunctive antecedents in counterfactuals (Fine, 1975; Alonso-Ovalle, 2009); alternative questions (Pruitt and Roelofsen, 2013); and phenomena of free choice (Aloni, 2007b; Menéndez-Benito, 2010; Aloni and Ciardelli, 2013). We will only discuss the latter here, but to do so we need first to specify an interpretation for the modal operator.

There are many ways to define a modal operator in a state-based system, we will only look at the following three:



(a) classical: $a \vee_{1/2} b$ (b) inquisitive: $a \vee_3 b$

Figure 3: Semantic content generated by different notions of disjunction

Definition 7 (Three notions of modality)

$s \models \Diamond_1\phi$	*iff*	$\forall w \in s : \exists t \subseteq R^{\rightarrow}(w) : t \neq \emptyset \;\&\; t \models \phi$	*("classical")*
$s \models \Diamond_2\phi$	*iff*	$s \not\models \neg\phi$	*(state-based)*
$s \models \Diamond_3\phi$	*iff*	$\forall w \in s : \forall t \in alt(\phi) : R^{\rightarrow}(w) \cap t \neq \emptyset$	*(alternative-sensitive)*

Auxiliary notions:
$R^{\rightarrow}(w) = \{v \mid wRv\}$ *&* $alt(\phi) = \{s \mid s \models \phi \;\&\; \neg\exists s' : s' \models \phi \;\&\; s \subset s'\}$.

The first notion, $\Diamond_1$, is a "classical" modal operator interpreted with respect to a relational structure (as for example in Humberstone 1981).

State-based $\Diamond_2$ has been proposed specifically for epistemic modals (Veltman 1996, Hawke & Steinert-Threlkeld 2016)[8] and is motivated by what Yalcin (2007) called *epistemic contradictions*, namely the illegitimacy of asserting both "it might be that ϕ" and "it is not the case that ϕ" in a single context, as illustrated by the unacceptability of examples like (14):

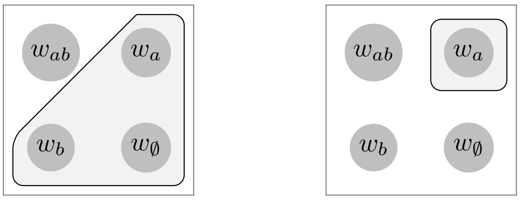
(14) #It might be raining but it is not raining.

By adopting $\Diamond_2$, which operates directly on s rather than on a set of R-accessible worlds, we derive epistemic contradictions ($\Diamond_2\phi \wedge \neg\phi \models \bot$) while preserving the non-factivity of the $\Diamond$ operator ($\Diamond_2\phi \not\models \phi$).

The alternative-sensitive $\Diamond_3$ is specifically motivated by FC phenomena (Aloni 2007, Ciardelli & Aloni 2013/16). The interpretation of $\Diamond_3$ is sensitive to the alternatives introduced in its scope, $\Diamond_3\phi$ is supported in s if for every $w \in s$, the set of worlds accessible from w is compatible with every alternative introduced by ϕ. If ϕ is inquisitive, it generates free choice effects. Otherwise, $\Diamond_3$ behaves classically, e.g., it does not give rise to "modal contradiction" ($\Diamond_3\phi \wedge \neg\phi \not\models \bot$) and it is not factive ($\Diamond_3\phi \not\models \phi$).

Classical $\Diamond_1\phi$ and alternative-sensitive $\Diamond_3\phi$ are distributive, but state-based $\Diamond_2\phi$ is not. Recall that for distributive sentences, our first two notions of disjunctions were equivalent while disjunction 3 was stronger. With the introduction of non-distributive formulas all logical connections between our three notions of disjunction disappear. Using the second notion of modality we can prove the following facts (the relevant counterexamples are illustrated in Figure 4):

[8] The definition adopted here is from Hawke & Steinert-Threlkeld (2016) and makes different prediction wrt embedded cases and treatment of the dual "must" than the original dynamic version of Veltman. Thanks to Daniel Rothschild and Simon Goldstein for discussion on these two points.



(a) $\models\ \Diamond_2 a \vee_2 \Diamond_2 b$, but $\not\models \Diamond_2 a \vee_1 \Diamond_2 b$

(b) $\models\ \Diamond_2 a \vee_1 \Diamond_2 b$, but $\not\models\ \Diamond_2 a \vee_2 \Diamond_2 b$

Figure 4: $\vee_1$ vs $\vee_2$ with non-distributive $\Diamond_2$

Fact 7 $\phi \vee_2 \psi \not\models \phi \vee_1 \psi$
Counterexample: $\{w_a, w_\emptyset, w_b\} \models \Diamond_2 a \vee_2 \Diamond_2 b$, *but* $\{w_a, w_\emptyset, w_b\} \not\models \Diamond_2 a \vee_1 \Diamond_2 b$

Fact 8 $\phi \vee_{1/3} \psi \not\models \phi \vee_2 \psi$
Counterexample: $\{w_a\} \models \Diamond_2 a \vee_{1/3} \Diamond_2 b$, *but* $\{w_a\} \not\models \Diamond_2 a \vee_2 \Diamond_2 b$

State (b) in Figure 4 does not support $\Diamond_2 a \vee_2 \Diamond_2 b$ because no subset of the state supports the second disjunct $\Diamond_2 b$. The empty set does not help here, type 2 possibility statements like $\Diamond_2 b$ require a non-empty state to be supported. This feature of $\Diamond_2$ can be crucially exploited to derive wide scope free choice inference (Hawke & Steinert-Threlkeld, 2016). Let's turn finally to the predictions concerning free choice.

It is easy to see that $\vee_1$ combined with $\Diamond_1$ fails to generate any free choice effect and in this sense this combination behaves like classical modal logic:

$$\Diamond_1 a \vee_1 \Diamond_1 b \quad \not\models \quad \Diamond_1 a \wedge \Diamond_1 b$$
$$\Diamond_1 (a \vee_1 b) \quad \not\models \quad \Diamond_1 a \wedge \Diamond_1 b$$

Assertability $\vee_2$ combined with state-based $\Diamond_2$ instead gives us wide scope FC effects (Hawke & Steinert-Threlkeld 2016) but does not generate narrow scope FC:

$$\Diamond_2 a \vee_2 \Diamond_2 b \quad \models \quad \Diamond_2 a \wedge \Diamond_2 b$$
$$\Diamond_2 (a \vee_2 b) \quad \not\models \quad \Diamond_2 a \wedge \Diamond_2 b$$

Inquisitive $\vee_3$ with alternative-sensitive $\Diamond_3$ gives us the opposite effect: narrow scope FC inference is generated, but no wide scope FC effects are derived (Aloni 2007, Ciardelli & Aloni 2013/16):

$$\Diamond_3 (a \vee_3 b) \quad \models \quad \Diamond_3 a \wedge \Diamond_3 b$$
$$\Diamond_3 a \vee_3 \Diamond_3 b \quad \not\models \quad \Diamond_3 a \wedge \Diamond_3 b$$

Thus, so far while none of the existing combinations accounts for both wide and narrow scope FC. Furthermore, whenever FC inducing sentences occur under negation, these systems predict weaker readings than attested in ordinary language use (see example (4) above):

$$\neg(\Diamond_2 a \vee_2 \Diamond_2 b) \quad \not\models \quad \neg\Diamond_2 a \wedge \neg\Diamond_2 b$$
$$\neg\Diamond_3 (a \vee_3 b) \quad \not\models \quad \neg\Diamond_3 a \wedge \neg\Diamond_3 b$$

In the next section, I will introduce a state-based system adopting an enriched version of $\vee_2$, which when combined with classical $\Diamond_1$, derives both wide and narrow scope FC while solving the "negation problem". Before doing so I need an argument of why deriving both wide and narrow scope FC is desirable.

Most existing accounts of free choice derive only one among wide scope and narrow scope FC and reduce the other case to a version of the derived case (terminology is adapted from Steinert-Threlkeld, 2017a, Appendix B):

- *Wide reductionism (narrow ⇒ wide)*: provides a treatment of wide scope free choice and narrow scope FC is reduced to wide scope FC: Zimmermann (2000), Steinert-Threlkeld (2017a)
- *Narrow reductionism (wide ⇒ narrow)*: provides a treatment of narrow scope free choice and wide scope FC is reduced narrow scope FC: e.g., Simons (2005), Fox (2007)

The system I will present in the next section instead is an example of a non-reductionist approach, providing a treatment of free choice which generates the inferences for both wide and narrow scope disjunctions (see also Starr, 2016).

In the literature a number of convincing arguments have been given against both reductionist strategies, I will mention here only two such arguments.

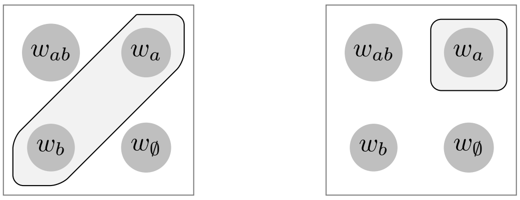
Problem for wide reductionism The following argument is from Fox (2007) (a proponent of narrow reductionism). Example (15) gives rise to both free choice inference and scalar implicature, but the derivation of the latter appears to require a narrow-scope disjunction analysis for the example (at least if we want to employ standard techniques):

(15) Mary may have ice-cream or cake. (+fc, narrow-scope)
   a. logical form: $\Diamond(a \vee b)$ / $\#\Diamond a \vee \Diamond b$
   b. free choice inference: $\Diamond a \wedge \Diamond b$
   c. scalar implicature: $\neg\Diamond(a \wedge b)$

Problem for narrow reductionism The following argument is from Alonso-Ovalle (2006). Example (16) gives rise to a free choice inference, but an analysis of (16) as a narrow scope disjunction would require dubious syntactic operations (e.g., Simons' (2005) covert across-the-board (ATB) movement of the modal would not work here, because ATB movement requires identical modals in each clause):

(16) You may email us or you can reach the Business License office at 949 644-3141. (+fc, wide-scope)
   a. logical form: $\Diamond a \vee \Diamond b$ / $\#\Diamond(a \vee b)$
   b. free choice inference : $\Diamond a \wedge \Diamond b$
   c. (no scalar implicature)

The last two arguments, when taken together, give us enough support for a non-reductionist approach.



(a) $\models (a \vee_+ b)$ (b) $\not\models (a \vee_+ b)$

Figure 5: Enriched disjunction

4 A state-based semantics for free choice

While all human languages appear to contain a word for negation, there are various examples of languages lacking explicit coordination structures. In these languages there is no word corresponding to *or*, but disjunctive meanings can typically still be expressed for example by adding a suffix/particle expressing uncertainty to the main verb. Example (17) illustrates this strategy for Maricopa (a Yuman language of Arizona described by Gill 1991):

(17) Johnš Billš v?aawuumšaa.
John-nom Bill-nom 3-come-pl-fut-infer
'John or Bill will come'

(18) Johnš Billš v?aawuum.
John-nom Bill-nom 3-come-pl-fut
'John and Bill will come' [Maricopa, Gil 1991, p. 102]

In (17) the "uncertainty" suffix *šaa* is added to the main verb and it is what triggers a disjunctive interpretation. Indeed when omitted as in (18) the interpretation of the sentence becomes conjunctive.

The state-based system I will present in this section takes very seriously the epistemic nature of disjunction illustrated by example (17): plain disjunctions will convey that the speaker is uncertain about which of the two disjuncts is true, as in Zimmermann (2000).[9]

More specifically, *or* will be analysed via an enriched version of $\vee_2$, which we will denote by $\vee_+$. A state s supports an **enriched disjunction** $(\phi \vee_+ \psi)$ iff s can be split into two *non-empty* substates, each supporting one of the disjuncts. E.g., as illustrated in Figure 5, the state $\{w_a, w_b\}$ (and $\{w_{ab}\}$) will support $(a \vee_+ b)$; but $\{w_a\}$ will no longer support $(a \vee_+ b)$, because no non-empty substate of $\{w_a\}$ supports the second disjunct. The latter fact will be crucial for the account of narrow scope FC.

To derive the fact that FC inferences do not embed under **negation** we will adopt a bilateral system (see Willer, 2018; Steinert-Threlkeld, 2017a; Hawke and Steinert-Threlkeld, 2017, for the same solution to the "negation problem"). The semantics will consist of a simultaneous recursive definition of the following two notions:

- $s \vdash \phi$ interpreted as "ϕ is assertable in s";
- $s \dashv \phi$ interpreted as "ϕ is rejectable in s".

[9]See also Harris' (2017) notion of a strong disjunction.

Modals are interpreted extending on the "classical" notion $\Diamond_1$. The differences between deontic and epistemic modals, for example with respect to epistemic contradictions, is captured in terms of properties of the accessibility relation R. Epistemic modals will be interpreted with respect to a so called *state-based* accessibility relation. Interpretation of deontic modals will not come with such requirement, only in some of their uses (including performative ones) the relevant accessibility relation will satisfy a property which, following Groenendijk, we will call *indisputability*.

Here is an outlook of the results: narrow scope FC will be always derived because in the interpretation of $\Diamond(\phi \vee_+ \psi)$ the relevant embedded state will have to support an enriched disjunction; wide scope FC will be derived, but only for R indisputable. Since epistemic modals are assumed to require a state-based R, and any state-based R is also indisputable, wide scope FC is always predicted for epistemic modals but not for deontic modals. The latter give rise to wide scope FC inference only in contexts satisfying indisputability. For R state-based, we will also derive epistemic contradictions, which will be so correctly predicted to arise only in the modal is read epistemically.

4.1 Definitions

The target language L is the language of propositional modal logic enriched with a constant NE which will be used to syntactically define the enriched version of disjunction $\vee_+$ and the $\bot$ operator:

Definition 8 (Language)

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Diamond\phi \mid \text{NE}$$

where $p \in A$.

The following abbreviations apply:

- $(\phi \vee_+ \psi) =: (\phi \wedge \text{NE}) \vee (\psi \wedge \text{NE})$
- $\bot =: \neg\text{NE}$
- $\Box\phi =: \neg\Diamond\neg\phi$

A model for L is a quadruple $M = \langle s_M, W, R, V\rangle$, where s_M is a subset of W, W is a set of worlds, R is an accessibility relation and V is a world-dependent valuation function for A. As in pointed Kripke models for classical modal logic, our models have a designated element, which here is a state, s_M , rather than a world. On the intended interpretation of these structures s_M stands for the information state of the relevant speaker.

Using the designated state s_M we can define the following state-based constraints on an accessibility relation R in a model M. Let $R^{\rightarrow}(w) = \{v \mid wRv\}$:[10]

- R is *indisputable* in M iff $\forall w, v \in s_M : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
- R is *state-based* in M iff $\forall w \in s_M : R^{\rightarrow}(w) = s_M$

[10]Indisputability relates to Zimmermann's (2000) Authority Principle, the property of being state-based relates to his Self-Reflection Principle.

An accessibility relation R is indisputable in a model M if any two worlds in s_M access exactly the same set of worlds according to R. Since s_M represents the information states of the relevant speaker, on this interpretation, an indisputable R means that the speaker is fully informed about R, so, for example, if R represents a deontic accessibility relation, indisputability means that the speaker is fully informed about what is obligatory or allowed.

An accessibility relation R is state-based in a model M if all and only worlds in s_M are accessible within s_M. Trivially is R is state-based, R is also indisputable. The adoption of a state-based R will lead to the satisfaction of the classical S5 axioms but it will also lead to an account of epistemic contradictions. For this reason we will assume a state-based R for epistemic modals but not for deontic ones:

- Epistemics: R is state-based
- Deontics: R is possibly indisputable (e.g. in performative uses)

Formulas in our language are interpreted in models M with respect to a state $s \subseteq W$ (not necessarily the designated state s_M). Both support and rejection conditions are specified:

Definition 9 (Semantic clauses)

$$
\begin{array}{rll}
M, s \vdash p & \textit{iff} & \forall w \in s : V(w,p) = 1 \\
M, s \dashv p & \textit{iff} & \forall w \in s : V(w,p) = 0 \\
M, s \vdash \neg\phi & \textit{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \textit{iff} & M, s \vdash \phi \\
M, s \vdash \phi \wedge \psi & \textit{iff} & M, s \vdash \phi \ \&\ M, s \vdash \psi \\
M, s \dashv \phi \wedge \psi & \textit{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi \\
M, s \vdash \phi \vee \psi & \textit{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \vdash \phi \ \&\ M, t' \vdash \psi \\
M, s \dashv \phi \vee \psi & \textit{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi \\
M, s \vdash \Diamond\phi & \textit{iff} & \forall w \in s : \exists t \subseteq R^{\rightarrow}(w) : t \neq \emptyset \ \&\ t \vdash \phi \\
M, s \dashv \Diamond\phi & \textit{iff} & \forall w \in s : \forall t \subseteq R^{\rightarrow}(w) : t \neq \emptyset \ \Rightarrow\ t \dashv \phi \\
M, s \vdash \text{NE} & \textit{iff} & s \neq \emptyset \\
M, s \dashv \text{NE} & \textit{iff} & s = \emptyset
\end{array}
$$

In the definition of logical consequence, we restrict attention to designated states s_M.

Definition 10 (Logical consequence) $\phi \models \psi$ *iff* $\forall M : M, s_M \vdash \phi \ \Rightarrow\ M, s_M \vdash \psi$

4.2 Predictions

Epistemic contradiction The first result is that we can derive epistemic contradiction, if R is state-based, while preserving the non-factivity of the possibility modal:

1. $\Diamond a \wedge \neg a \models \bot$ [if R is state-based]

2. $\Diamond a \not\models a$ [even if R is state-based]

As already mentioned above, in this system the difference between different modalities is captured in terms of differences in properties of the accessibility relation. Assuming a state-based R for epistemic modals but not for deontic modals, we correctly predict that only the former lead to epistemic contradiction effects:

(19) #It might be raining and it is not raining.

(20) You are not there but you may go there.

The assumption that epistemic modals trigger state-based R has consequences also for their free choice potential.

Free choice We derive both narrow scope and wide scope FC effects by adopting the enriched version of disjunction, but while narrow scope effects are generated for any kind of modality, wide scope FC arises only in case the modality is of the indisputable kind.

1. $\Diamond(a \vee_+ b) \models \Diamond a \wedge \Diamond b$

2. $\Diamond a \vee_+ \Diamond b \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Since state-based R are also indisputable, narrow and wide scope FC are always predicted for epistemics, which involve state-based R:

(21) He might either be in London or in Paris. [+fc, narrow]

(22) He might be in London or he might be in Paris. [+fc, wide]

The case of deontic FC is more subtle. Assuming that deontics trigger an indisputable R only in certain contexts, namely when the speaker is assumed to be knowledgable about what is permitted/obligatory (e.g. in performative uses), we make the following predictions:

- narrow scope FC always predicted for deontics
- wide scope FC predicted only if speaker knows what is permitted/obligatory

These predictions have received preliminary confirmations from recent experiments reported in Cremers *et al.* (2017). In these experiments, judgements on free choice effects were collected, for both wide and narrow scope disjunctions, in different contexts with the speaker assumed to be knowledgable or not. To distinguish narrow scope from wide scope configurations, examples like the following were used, where the position of *either* arguably constrains the syntactic scope of *or*:

(23) We may either eat the cake or the ice-cream. [narrow scope disjunction favoured, but not forced]

(24) Either we may eat the cake or the ice-cream. [wide scope disjunction forced]

More specifically, as argued by Larson (1985), the high position of *either* in (24) forces a wide scope disjunction configuration, while its low position in (23) favours a narrow scope interpretation. One rather surprising result of these experiments was that only in wide scope configurations like (24) the availability of FC inference was dependent on the assumption on speaker knowledge, exactly as predicted by the present analysis.

A further consequence of our analysis is that all cases of overt free choice cancellation must be treated as examples of wide scope disjunction. This is arguably the case for examples like (25) involving sluicing, as discussed in Fusco (2018):

(25) You may either eat the cake or the ice-cream, I don't know which.

Whether the same assumption is also justified in cases like (26), discussed by Kaufmann (2016), must be left to another occasion:

(26) You may either eat the cake or the ice-cream, it depends on what John has taken.

If we assume that sluicing always requires disjunctions as antecedents, the first sentence in (27) will also be a case of a wide scope disjunction:

(27) You may either eat the cake or the ice-cream, I don't care which.

But then we predict that (27) (i) generate free choice inferences only in contexts where the speaker is assumed to be knowledgable and (ii) never triggers not-both scalar implicature of the sort $\neg\Diamond(a \wedge b)$, which would require narrow scope configurations (see discussion around example (15)). These are testable predictions which would be interesting to further investigate.

In what follows we discuss more facts and some problems.

Plain disjunction and necessity The present system derives FC effects also for plain disjunctions, if R is state-based, and under the necessity operator $\Box$:

- $a \vee_+ b \models \Diamond a \wedge \Diamond b$ [if R is state-based]
- $\Box(a \vee_+ b) \models \Diamond a \wedge \Diamond b$ ($\Box \equiv \neg\Diamond\neg$)

The first fact has been argued to be problematic by Hawke and Steinert-Threlkeld (2016). To avoid this inference we could make the insertion of NE somehow triggered by the modal rather than by 'or'. Ideally, however, we would like to formulate general (conversational) principles regulating the distribution of NE rather than postulating its presence in certain constructions (see also below). The investigation of such principles however must be left to future work.

Negation It is easy to see that all predicted free choice effects correctly disappear under negation:

- $\neg\Diamond(a \vee_+ b) \models \neg\Diamond a \wedge \neg\Diamond b$
- $\neg(\Diamond a \vee_+ \Diamond b) \models \neg\Diamond a \wedge \neg\Diamond b$
- $\neg(a \vee_+ b) \models \neg a \wedge \neg b$

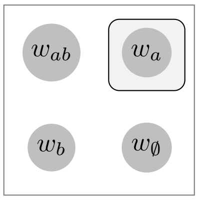


Figure 6: $\{w_a\} \models \neg_1(a \vee_+ b)$, but $\{w_a\} \not\models \neg_2(a \vee_+ b)$

Behaviour under negation, however, is postulated here rather than predicted. Allowing to pre-encode what should happen under negation, bilateral systems like ours are more descriptive than explanatory.

Alternatively we could adopt a unilateral system and explain the negation facts by either (i) treat *or* as ambiguous between $\vee_+$ and $\vee$ and adopt a principle like the strongest meaning hypothesis to explain why enriched disjunction readings are never selected under negation (e.g., Aloni, 2007b); or (ii) treat *or* as $\vee$ but formulate general principles regulating the distribution of NE, i.e. deriving its default insertion in positive contexts and its exclusion from downward entailing contexts (roughly NE as a positive polarity item).

It's interesting to see that in such a unilateral system, different results would obtain if we adopt different notions of negation:

- $s \models \neg_1\phi$ iff $\forall t \subseteq s : t \models \phi \Rightarrow t = \emptyset$ [intuitionistic]
- $s \models \neg_2\phi$ iff $s \cap t = \emptyset$, for all $t : t \models \phi$ [incompatibility]

For example, the following obtains as illustrated in Figure 6:

- $\neg_1(a \vee_+ b) \not\models \neg_1 a$
- $\neg_2(a \vee_+ b) \models \neg_2 a$

Although from a conceptual point of view unilateral systems are attractive, an empirical argument for a bilateral account of narrow scope free choice has been recently proposed by Romoli and Santorio (p.c.). Romoli and Santorio consider examples like (28).

(28) a. Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go to study in Japan (and the second who can go to study in the States).
b. $\neg\Diamond(a \vee_+ b) \vee \phi_{\Diamond a}$

In (28), the second disjunct *She is the first in our family who can go to study in Japan* presupposes *She can go to study in Japan*, but this presupposition does not project, it is filtered by the negation of the first disjunct. Assuming that a disjunction $\phi \vee \psi_P$ presupposes $\neg\phi \rightarrow P$, the predicted presupposition for (28) is (29):

(29) $\neg\neg\Diamond(a \vee_+ b) \rightarrow \Diamond a$

In bilateral accounts of narrow scope FC like Willer (2018) and the present system, (29) is a tautology (double negations cancel each other out and free choice inference is computed), so the correct filtering is predicted. Unilateral systems, which typically do not generate FC effects under double negation, will have to come up with an alternative explanation of these facts.

Before concluding I will discuss two more problems arising for the present analysis:

Zimmermann's problem The following (possibly less worrying) version of a problem which arose for Zimmermann 2000 arises here as well (see Geurts, 2005):

(30) $\Box a \vee_{+} \Box b \models \Box a \wedge \Box b$ [if R is indisputable]

(30) means that any felicitous case of disjunction of necessities like (31) must be treated in the present system as a case of narrow scope disjunction, $\Box(a \vee b)$, (possibly using Simons' (2005) ATB movement) or as a case where indisputability is not satisfied.

(31) You must invite John or you must invite Mary.

This is not necessarily a bad prediction. Uncontroversial wide scope disjunctions of necessity statements like (32) are arguably only felicitous in contexts where the speaker is uncertain about what must be the case and these are typically cases which would require an R which is *not* indisputable:

(32) Either you must invite John or you must invite Mary.

Mandelkern's problem In the present system disjunctions of epistemic contradictions are not contradictory and this is a wrong prediction (Mandelkern, 2017):[11]

(33) $(a \wedge \Diamond\neg a) \vee (b \wedge \Diamond\neg b) \not\models \bot$ [even if R is state-based]

(34) #Jo isn't tall but she might be, or Jim isn't tall but he might be.

To solve Mandelkern's problem we could go back to a more Veltman-like treatment of epistemic modals, where the modal operates on the local context and not on a set of accessible worlds:

$$M, s \vdash \Diamond_x \phi \quad \text{iff} \quad \forall w \in s : \exists t \subseteq s : t \neq \emptyset \ \& \ t \vdash \phi$$
$$M, s \dashv \Diamond_x \phi \quad \text{iff} \quad \forall w \in s : \forall t \subseteq s : t \neq \emptyset \ \Rightarrow \ t \dashv \phi$$

Note however that the adoption of such a local notion of epistemic modality would lead to other counterintuitive results in combination with our notion of disjunction and negation. For example $\neg\Diamond_x a \vee_{+} \neg\Diamond_x b$ would be compatible with $\Diamond_x a \wedge \Diamond_x b$ as illustrated in Figure 7 unless we adopt a different rejection clause for the modal (this needs to be checked, Hawke and Steinert-Threlkeld (2016) had a similar problem. If I remember correctly in their more recent work they were leading towards a solution in this direction: a local notion of modality with a non-standard rejection clause).

[11] Besides Mandelkern (2017) the only other analysis of epistemic contradictions that does not have this problem is, as far as I know, the one defended in Hawke and Steinert-Threlkeld (2016); Steinert-Threlkeld (2017a,b); Hawke and Steinert-Threlkeld (2017).

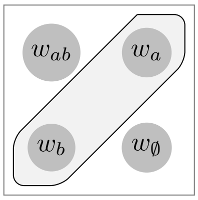


Figure 7: A state supporting both $\neg\Diamond_x a \vee_+ \neg\Diamond b$ and $\Diamond_x a \wedge \Diamond_x b$

Alternatively, we could adopt the following notion of epistemic modality and disjunction in a double indexed version of the semantics. I only give here unilateral entries because the rejection clauses do not play any role (this is an elaboration of a suggestion by Wes Holliday, all details need to be checked, all mistakes are mine):

$$
\begin{array}{lll}
M,t,s \models \Diamond_e\phi & \text{iff} & \exists t' \subseteq t : t' \neq \emptyset \;\&\; M,t,t' \models \phi \\
M,t,s \models \phi \vee \psi & \text{iff} & \exists s_1, s_2 : s_1 \cup s_2 = s \;\&\; M,t,s_1 \models \phi \;\&\; M,t,s_2 \models \psi \\
 & & \&\; M,s_1,s_1 \models \phi \;\&\; M,s_2,s_2 \models \psi
\end{array}
$$

The issue which of these two solutions is preferable must be left to another occasion.

5 Summary and further work

We compared three notions of disjunction in a state-based semantics ($\vee_1$ from possibility/dynamic semantics; $\vee_2$ from team/assertability logic; and $\vee_3$ from inquisitive/truthmaker semantics) and then proposed a state-based logical account of FC inference using an enriched version of $\vee_2$. In the defined semantics, narrow and wide scope FC inference are derived as entailments (well-behaving under negation) with wide scope FC dependent on the nature of the accessibility relation. Although the semantics validates a number of classical laws (double negation, De Morgan) the logic is highly non-standard, for example, only a restricted version of the introduction of disjunction rule will hold ($\phi \models \phi \vee \psi$ only if NE does not occur in ψ). A full axiomatisation is left for future work as well as the exploration of the dynamic potential of the system and its extension to the first order case. In future work we would further like to (i) investigate experimentally the predictions of the system with respect to the differences between wide/narrow scope epistemic and deontic FC; (ii) study the formal properties and interaction of different kinds of modals (deontics, epistemics, etc) and how these could relate to different constraints on the accessibility relation in our system; (iii) integrate in the current semantics the state-based pragmatics discussed in Aloni & Franke (2012) to arrive at a system where different kinds of inferences (automatised free choice, scalar implicatures, fossilised (embeddable) implicatures) can be derived and their interaction can be studied.

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