

Greek letters will *only* be expressions in our metalanguage (that designate strings/expressions of our object language).

1. alpha	= "α"	Alpha	= "Α"
2. beta	= "β"	Beta	= "Β"
3. gamma	= "γ"	Gamma	= "Γ"
4. delta	= "δ"	Delta	= "Δ"
5. epsilon	= "ε"	Epsilon	= "Ε"
6. zeta	= "ζ"	Zeta	= "Ζ"
7. eta	= "η"	Ēta	= "Η"
8. theta	= "θ"	Theta	= "Θ"
9. iota	= "ι"	Iota	= "Ι"
10. kappa	= "κ"	Kappa	= "Κ"
11. lambda	= "λ"	Lambda	= "Λ"
12. mu	= "μ"	Mu	= "Μ"
13. nu	= "ν"	Nu	= "Ν"
14. xi	= "ξ"	Xi	= "Ξ"
15. omicron	= "ο"	Omicron	= "Ο"
16. pi	= "π"	Pi	= "Π"
17. rho	= "ρ ϱ"	Rho	= "Ρ"
18. sigma	= "σ ς"	Sigma	= "Σ"
19. tau	= "τ"	Tau	= "Τ"
20. upsilon	= "υ"	Upsilon	= "Υ"
21. phi	= "φ"	Phi	= "Φ"
22. chi	= "χ"	Chi	= "Χ"
23. psi	= "ψ"	Psi	= "Ψ"
24. omega	= "ω"	Omega	= "Ω"

Our conventions:

* using capital Greek letters as variables in the metalanguage ("metavariables") designating **sets of** object language **formulas**

* using small Greek letters like ϕ (phi), ψ (psi), χ (chi) as metavariables designating **single** object language **formulas**

* using small Greek letters like α , β as metavariables designating **term expressions** of the object language

* using small Greek letters like ξ (xi), ζ (zeta) as metavariables designating **variables** in the object language (x , x' , x'' , x''' , etc.)

Also only part of the metalanguage: spelled-out words like "and", "or", "iff" (the object language will use symbols)

When the signature of the object language is meant to represent numbers, I'll *try* to stop using 0 as a term constant in the object language, instead I'll use Z. I'll use S as a functor expression in the object language, so the idea will be that these object language expressions are *meant to* be standing for these objects (numbers) in a model's domain:

Z --> 0
SZ --> 1
SSZ --> 2
SSSZ --> 3
etc.

When n is a *metalanguage* variable for a number (≥ 0), I'll sometimes write $S^n Z$ to stand for the object language term expression consisting of n copies of S followed by a Z. This is a piece of metalanguage notation for designating that object language string. The n and the superscripts aren't grammatical inside the object language.

When we write just bare Z, SZ, and so on, these are our metalanguage names for those same object language strings.

In some ways it'd be helpful if we also distinguished between the use of symbols like +, =, < inside the object language and inside the metalanguage. Sometimes I'll write out the words "plus" and so on; that will always be meant as part of the metalanguage. But inevitably sometimes we'll also use the symbols in the metalanguage. If it's important to be very explicit, I might write something like this:

$S_n Z \hat{=} "+" \hat{=} SZ$

but if/when I write:

$S^n Z + SZ$

you should understand it the same way. It doesn't make sense for the + in the second line to be the metalanguage plus symbol, since the expressions before it and after it are expressions designating *strings in the object language*, not expressions designating numbers. That whole metalanguage expression picks out a complex object language term expression.

If, on the other hand, I write $n + 1$, then I am using + to mean addition, and this metalanguage expression designates a number.

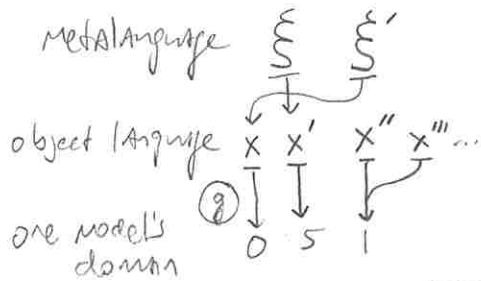
Moby Dick



The first letter of Moby Dick is "C".
(using)

The first letter of Moby Dick is "M".
(mentioning)

Assignments $g(\cdot)$ and $\pi(\cdot)$ take $\langle \text{metalinguage expressions designating object language strings} \rangle$ as arguments, deliver objects from the model's domain (or sets/functions over them) as results



What is $g(\xi)$?

What is $g(\xi^{x''})$?

$\Gamma \models \phi$
entails
has as logical
consequence

$\Gamma, \Delta, \Psi \models \phi$

$\models \phi$
is logically true
valid formula/sentence

Some logical formulas are called
Tautologies

$0=0 \vee 0 \neq 0$ } yes
 $0=0$ } no
 $\exists x(x=0)$ } no

$0 \neq 1$ } not every
 $0 < 1$ } logical formulas

When ϕ is true or
satisfied by
some model (+ ass)

$\models \phi$ is satisfiable

set Γ is satisfiable
when there's some
model making all of its
elements true

When ϕ is true or
satisfied by no model (+ ass)

$\models \phi$ is unsatisfiable
logically false

set Γ is unsatisfiable when
there's no model making all
of its elements true

Can write as: $\Gamma \models \perp$

Some logical falsehoods are called
Contradictions

$0=0 \wedge 0 \neq 0$ } yes
 $0 \neq 0$ } no

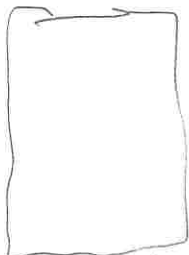
$\phi \models \psi$ means ϕ, ψ are logically equivalent
meaning for every model + ass (M, g) $\models \phi \models_M g = \models \psi \models_M g$
equivalent to
 $\phi \models \psi$ and $\psi \models \phi$

ϕ is unsatisfiable iff $\sim \phi$ is true
on every model (+ ass)

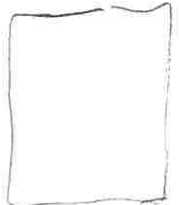
$\Gamma \cup \{ \phi \}$ is unsat iff $\Gamma \models \sim \phi$
(What if the fault is inside Γ ?
 $P, \sim P \models$ anything)

(Goldrei uses \equiv ; others use other notation)

NEIGHBORHOODS

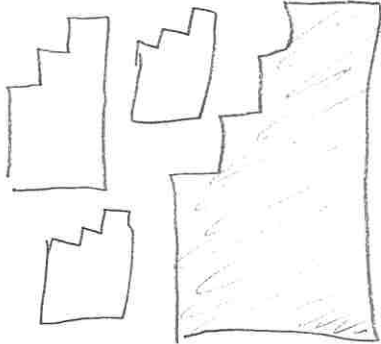


Intuitionistic logics



Paraconsistent & Relevance logics

Classical logics



one of the most well-equipped first-order predicate logic factories



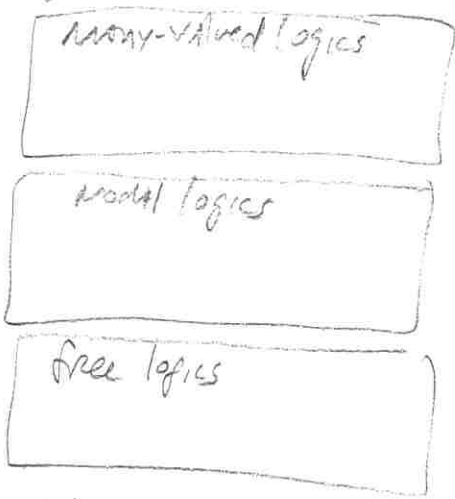
Factories: which resources do you make available (quantifiers? $=?$ \wedge and \vee can have factories like $+$, \cup ?)

can quantify over predicates or only objects?

predicates can apply to other predicates or only objects?

Which connectives: \neg , \supset , \perp , ... XOR, \leftrightarrow , ...?)

Extensions (st include all theories of classical logic, st not...)



many-valued logics

modal logics

free logics

etc...

Constraints

What non-logical vocab (signature) allowed

YOU SUPPLY

