11.4 Predicate Logic

Let us go back to an example from the beginning of the last chapter (now rewritten a bit more formally):

(For all x)(x is Australian → x likes cricket)
Mel Gibson is Australian

Mel Gibson likes cricket

As I said, this is a valid argument. But its validity isn't just a matter of truth-functional connectives. It also depends on the way that the argument involves *universal quantification*—'(For all x)(...)'.

Here is another example.

John is tall
John is fat

(There is an x such that)(x is tall and fat)

Here the validity of the argument depends on its use of *existential quantification*—'(There is an x such that)(...)'.

Predicate logic is concerned with arguments whose validity depends on universal and existential quantification, as well as on the truth-functional connectives. (We can think of predicate logic as *including* propositional logic but adding some further structure.)

Just as with propositional logic, we can analyse logical consequence in predicate logic both *syntactically* and *semantically*.

11.5 Predicate Syntax

To get a syntactic account of logical consequence for predicate logic, we need to add some extra rules of inference for the quantifiers to those for the truth-functional connectives. In particular, we need introduction and elimination rules for both the universal and existential quantifier.

These are a bit messy to state precisely, so let me just give the general idea.

To understand the elimination rule for universal quantification, note that what goes for everything goes for any particular thing. A rough version of the rule is thus:

Given a condition *F* and a name *a*, you can move from '(For all x)(x is *F*)' to '*a* is *F*'.

To understand the introduction rule for existential quantification, note that what goes for a particular thing goes for something. A rough version of the rule is thus:

Given a condition *F* and a name *a*, you can move from '*a* is *F*' to '(There is an x such that)(x is *F*)'.

The other two rules are a bit harder to grasp. The introduction rule for universal quantification says that:

If, given some condition *F*, you can prove '*a* is *F*' whatever name *a* is used, then you can move to '(For all x)(x is *F*)'.

(The idea is that a condition must apply to everything if there is a proof which can show it applies to any particular thing.)

And the elimination rule for existential quantification says that:

You can move from '(There is an x such that)(x is *F*)' to a sentence p, just in case p can be proved from 'a is *F*' whatever name *a* is used.

(The idea is that if p follows from an arbitrary object's satisfying a condition *F*, then it must follow from *something* satisfying *F*.)

Once we have specified a set of rules of inference, we can define notions of proof and syntactic consequence for predicate logic just as we did for propositional logic. A sentence j is a syntactic consequence of a set of sentences K in predicate logic just in case there is a proof in predicate logic with premises K and conclusion j. In such a case we write $K \vdash_{PRED} j$, and we say that j is *provable* from K in predicate logic. And if we can prove j from zero premises, we write $\vdash_{PRED}$ j and say that j is *provable* simpliciter, or that j is a *theorem* of predicate logic.

11.6 Predicate Semantics

Just as the syntax for predicate logic expands the syntax for propositional logic, so does its semantics expand propositional semantics. In the last chapter we saw how the truth values of sentences with truth-functional structure depend on the semantic values of their parts. Predicate semantics adds to this propositional semantics a further explanation of how the truth values of sentences with *quantificational* structure similarly depend on the semantic values of their parts.

To achieve this, we suppose that there is some set of objects at issue when we say '(For all x)(…)' or '(There is an x)(…)'. This set is called 'the domain of discourse'. We then further suppose that all names refer to some object in this domain, and that all predicates are associated with some subset of this domain.

We can then say that, given any name *a* and predicate *F*, a sentence of the form '*a* is *F*' will be true just in case the object named by *a* is a member of the set associated with *F*.

And we can also say that any sentence of the form '(For all x)(x is *F*)' will be true just in case everything in the domain of discourse is in the set associated with *F*.

Similarly, any sentence of the form '(There is an x)(x is *F*)' will be true just in case something in the domain of discourse is in the set associated with *F*.

(I am here skating over some technicalities that arise from the fact that quantified sentences can involve *complex* conditions constructed by applying truth-functional connectives to *predicates*, whereas so far we have only dealt with the semantic contribution of truth-functional connectives to complex *sentences*. But the above is already enough to give the general idea of predicate semantics.)

Again, once we are armed with a semantics for predicate logic, we can define a notion of semantic consequence for predicate logic just as we did for propositional logic. A sentence j is a semantic consequence of a set of sentences K in predicate logic just in case the semantics for predicate logic ensures that j must be true whenever the sentences in K are all true. In such cases we write $K \vdash_{\text{PRED}} j$, and we say that j is a *semantic* consequence of K in predicate logic. And if the semantics ensures that j will be true whatever is the case, then we write $\vdash_{\text{PRED}} j$, and we say that j is a *logical truth* in predicate logic.