4 Arguments and Inferences

4.1 Arguments and Argument Schemata

So far we have mainly been concerned with the truth of sentences. To this end we have constructed a formal language, that of predicate logic, and have shown how to translate (certain kinds of) natural language sentences into it. We have also developed conditions which determine the truth or falsity of given sentences in predicate logic under given circumstances, that is, in any given models. Not that we had any particular sentences in mind whose truth or falsity we wished to assess. Our idea was to show how the truth value of a sentence depends on the meanings of the parts from which it is built up.

We shall now turn to another, related matter: the ways that accepting certain sentences can commit one to accept other sentences. This is an important facet of the more general question of the interdependencies between the meanings of sentences.

It is quite common, in everyday language, to accept a sentence just because one has previously accepted certain other sentences from which it follows by some kind of argument. The simplest arguments are those in which a number of previously accepted sentences (the *assumptions*, or premises) are followed by an expression such as *thus* and then a new sentence (the argument's *conclusion*). We saw some examples of arguments in §1.1. In chapters 2 and 3 we translated sentences derived from natural language into formal language, and now we shall do the same for arguments. But we shall stick to these simple kinds of arguments, since so many factors determine the forms of arguments and the extent to which they are found convincing that a general treatment would seem still to be beyond our reach. You could say that we restrict ourselves in logic to the results which an argument yields, which is in a way another *extensionalization:* the only thing which really matters about an argument is whether or not its conclusion is justified by its assumptions. Translating the assumptions of a given argument into predicate logic as the sentences $\phi_1, \ldots, \phi_n$ and its conclusion as the sentence ψ, we obtain an *argument schema* $\phi_1, \ldots, \phi_n/\psi$. It has $\phi_1, \ldots, \phi_n$ as its premises and ψ as its conclusion. If accepting $\phi_1, \ldots, \phi_n$ commits one to accepting ψ, then this argument schema is said to be *valid* and ψ is said to be a *logical consequence* of $\phi_1, \ldots, \phi_n$. An informal argument is also said to be valid if it can be translated into a valid argument schema.

The logical constants appearing in the formulas of an argument schema are the only symbols whose meaning determines whether it is valid or not. This can result in some intuitively valid schemata being pronounced invalid, since expressions other than logical constants can hide aspects of meaning which lend arguments intuitive credibility. This can be avoided, for example, by making the hidden meanings explicit in additional premises. Indeed, we saw an example of just this in the discussion of argument (8) in §3.1, here renumbered as argument (1).

(1) Casper is bigger than John.
John is bigger than Peter.
―――――――――――――――
Casper is bigger than Peter.

A direct translation results in an argument schema which (as we shall see) is invalid: Bcj,Bjp/Bcp. But adding the transitivity of *bigger than*, mentioned in that discussion, results in the following argument schema, which (as we shall see) *is* valid: $Bcj, Bjp, \forall x\forall y\forall z((Bxy \wedge Byz) \rightarrow Bxz)/Bcp$.

There are two essentially different approaches to the notion of validity as it applies to argument schemata. The first of these is the *semantic* approach, which involves the interpretation of the sentences of predicate logic and thus concepts like models and truth. This approach will be developed systematically in §4.2, but it can do no harm to anticipate by giving the obvious definition of (semantic) validity for argument schemata in predicate logic.

Definition 1

$\phi_1, \ldots, \phi_n/\psi$ is *semantically valid* if for all models **M** which interpret all the predicate letters and constants and any function symbols appearing in $\phi_1, \ldots, \phi_n, \psi$ and for which $V_M(\phi_1) = \ldots = V_M(\phi_n) = 1$, we also have $V_M(\psi) = 1$.

In other words, $\phi_1, \ldots, \phi_n/\psi$ is (semantically) valid if it is not possible that both $V_M(\phi_1) = \ldots = V_M(\phi_n) = 1$ and $V_M(\psi) = 0$. Accepting the *truth* of $\phi_1, \ldots, \phi_n$ thus commits one to accepting the truth of ψ. Where $\phi_1, \ldots, \phi_n/\psi$ does not contain any premises, so that $n = 0$, the validity of the argument schema depends on whether or not ψ can be concluded anyway, from nothing at all. Then the definition reduces to: $/\psi$ is semantically valid iff ψ is universally valid (in propositional logic: a tautology).

The second line of approach to the notion of validity is via syntactic methods. Although semantic methods tend to give one a better understanding (and tend to be more fertile with regard to, for example, linguistic applications), no introduction to logic would be complete without a syntactic treatment of the notion of inference. The semantic notion of validity is based on universal quantification over that mysterious totality, the class of all models (there are infinitely many models, and models can themselves be infinitely large). The notion of meaning which we use in the syntactic approach is more instrumental: the meaning of some part of a sentence lies in the conclusions which, because precisely that part appears at precisely that place, can be drawn from that sentence. Against the background formed by such considerations, a very precise and finite list of small, almost entirely trivial steps of reasoning is drawn up. These steps can be linked to form the longer, formal chains of reasoning which are called *derivations*. Relations of syntactic inference are then of the form: $\phi_1, \ldots, \phi_n/\psi$ is *syntactically valid* iff there is a derivation of ψ from $\phi_1, \ldots, \phi_n$. The syntactic approach which we have chosen is that of *natural deduction*. It illustrates the instrumental point of view on the meaning of connectives and quantifiers most clearly. And this new point of view should also help to deepen our understanding of what the logical constants mean.

We will discuss the semantic and syntactic approaches in §§4.2 and 4.3, respectively. Then we will discuss important connections between the two in §4.4. It turns out that these two divergent methods ultimately lead to exactly the same argument schemata being pronounced valid. It is comforting to know that the semantic notion of validity, with its heavy ontological commitment, is parallel to simple combinatory methods which entirely avoid such abstract concepts (see §4.4).

We conclude this section with a few remarks on the connection between inference relations and the meaning of a sentence or a part of a sentence. Actually, the fact that, for example, ψ follows from ϕ (ϕ/ψ is valid) indicates a connection between the meanings of ϕ and ψ. But if not only does ϕ follow from ϕ but ϕ in turn follows from ψ, then there is a sense in which ϕ and ψ have the same meaning. In such cases ϕ and ψ are said to have the same *extensional meaning*. It is not too difficult to see (and it will be proved in theorem 3 in §4.2.2) that semantically speaking, this amounts to the equivalence of ϕ and ψ. Predicate logic has the property that ϕ and ψ can be freely substituted for each other without loss of extensional meaning as long as they are equivalent (i.e., as long as they have the same extensional meaning). We referred to this as *the principle of extensionality* for predicate logic. These remarks apply directly only to those sentences which share the same meaning in the strict, 'logical' sense. Pairs like (2) and (3) are a bit more complicated:

(2) Casper is bigger than Peter.
Peter is smaller than Casper.

(3) Pierre is a bachelor.
Pierre is an unmarried man.

We will discuss this briefly in §4.2.2.

4.2 Semantic Inference Relations

4.2.1 *Semantic validity*

Let us first review the definition of semantic validity, which we shall refer to simply as *validity*, in a slightly different manner. We give the definition for

predicate logic first; the obvious restriction to propositional logic follows immediately.

Definition 2

(a) A model **M** is *suitable* for the argument schema $\phi_1, \ldots, \phi_n/\psi$ if all predicate letters, constants, and function symbols appearing in $\phi_1, \ldots, \phi_n$ or in ψ are interpreted in **M**.
(b) $\phi_1, \ldots, \phi_n/\psi$ is said to be *valid* (shorter notation: $\phi_1, \ldots, \phi_n \models \psi$) if for every model **M** which is suitable for $\phi_1, \ldots, \phi_n/\psi$ and such that $V_M(\phi_1) = \ldots = V_M(\phi_n) = 1$, $V_M(\psi) = 1$.

In that case we also say that ψ *is a semantic consequence of* $\phi_1, \ldots, \phi_n$. If $\phi_1, \ldots, \phi_n/\psi$ is not valid, then this may also be written as $\phi_1, \ldots, \phi_n \not\models \psi$.

Note that the validity of $\phi_1, \ldots, \phi_n/\psi$ reduces to the universal validity of ψ if $n = 0$, and that the notation $\models$ is therefore no more than an expansion of the notation introduced in §3.6.4. The definition for propositional logic is slightly simpler:

Definition 3

For formulas $\phi_1, \ldots, \phi_n, \psi$ in propositional logic, $\phi_1, \ldots, \phi_n \models \psi$ holds just in case for all valuations V such that $V_M(\phi_1) = \ldots = V_M(\phi_n) = 1$, $V_M(\psi) = 1$.

We could of course restrict ourselves to valuations 'suitable' for $\phi_1, \ldots, \phi_n/\psi$, these being functions which map all the propositional letters appearing in $\phi_1, \ldots, \phi_n, \psi$ onto 0 or 1, but not necessarily all the others. In fact, that is more or less what is done in truth tables.

The validity of every argument schema in propositional logic can be decided by means of truth tables. We shall discuss schemata (4) and (5) as examples:

(4) $p \rightarrow (q \wedge r), q \rightarrow \neg r / \neg p$

(5) $\neg p \rightarrow (q \wedge \neg r), \neg q \rightarrow \neg r / p$

A truth table for (4) is given in (6):

(6)

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$\neg r$	$q \rightarrow \neg r$	/	$\neg p$
1	1	1	1	1	0	0		
1	1	0	0	0	1	1		
1	0	1	0	0	0	1		
1	0	0	0	0	1	1		
0	1	1	1	1	0	0		
0	1	0	0	1	1	1	*	1
0	0	1	0	1	0	1	*	1
0	0	0	0	1	1	1	*	1

We only have to consider the valuation of the conclusion $\neg p$ in those cases (marked with a *) in which the valuations of the premises $p \to (q \wedge r)$ and $q \to \neg r$ are both 1. Now $\neg p$ has the value 1 in each of these three cases. So $p \to (q \wedge r), q \to \neg r \models \neg p$.

The truth table for schema (5) is in (7):

(7)

p	q	r	$\neg p$	$\neg r$	$q \wedge \neg r$	$\neg p \to (q \wedge \neg r)$	$\neg q$	$\neg q \to \neg r$	/	p
1	1	1	0	0	0	1	0	1	*	1
1	1	0	0	1	1	1	0	1	*	1
1	0	1	0	0	0	1	1	0		
1	0	0	0	1	0	1	1	1	*	1
0	1	1	1	0	0	0	0	1		
0	1	0	1	1	1	1	0	1	*	0
0	0	1	1	0	0	0	1	0		
0	0	0	1	1	0	0	1	1		

From the truth table it is apparent that if V is such that $V(p) = 0$, $V(q) = 1$, and $V(r) = 0$, then $V(\neg p \to (q \wedge \neg r)) = V(\neg q \to \neg r) = 1$ and $V(p) = 0$ hold for V. From this it is clear that $\neg p \to (q \wedge \neg r), \neg q \to \neg r \not\models p$. A valuation like V with $V(p) = 0$, $V(q) = 1$, and $V(r) = 0$ which shows that an argument schema is not valid is called a *counterexample* to that argument schema. (The given V is a counterexample to $\neg p \to (q \wedge \neg r), \neg q \to \neg r/p$, for example.)

Such a counterexample can always be turned into a real-life counterexample if one wishes, by replacing the propositional letters by actual sentences with the same truth values as the propositions they replace. In this case, for example:

p: New York is in the United Kingdom; q: London is in the United Kingdom; r: Moscow is in the United Kingdom.

Exercise 1

Determine whether the following argument schemata are valid. If a schema is invalid, give a counterexample.

(a) $p \wedge q/p$
(b) $p \wedge q/q$
(c) $p \vee q/p$
(d) $p, q/p \wedge q$
(e) $p/p \vee q$
(f) $q/p \vee q$
(g) $p/p \wedge q$
(h) $p, p \to q/q$
(i) $p, q \to p/q$
(j) $p, \neg p/q$
(k) $p \to (q \wedge \neg q)/\neg p$
(l) $p \vee q, p \to r, q \to r/r$
(m) $p \vee q, (p \wedge q) \to r/r$
(n) $p \vee q, p \to q/q$
(o) $p \vee q, p \to q/p$
(p) $p \to q, \neg q/\neg p$
(q) $p \to q/\neg p \to \neg q$

One essential difference between propositional logic and predicate logic is this: some finite number of (suitable) valuations will always suffice to determine the validity of an argument schema in propositional logic, whereas an infinite number of models can be relevant to the validity of an argument schema in predicate logic; and the models can themselves be infinite as well. This suggests that there may well be no method which would enable us to determine in a finite number of steps whether any given argument schema in predicate logic is valid or not. The suspicion that no general method exists has been given a precise formulation and has been proved; this is surely one of the most striking results in modern logic (see Church's Theorem, §4.4). There are systematic methods for investigating the validity of argument schemata in predicate logic, incidentally, but these cannot guarantee a positive or negative result within a finite time for every argument schema. We will not discuss any of these systematic methods but will give a few examples which show that in practice things are not so bad as long as we stick to simple formulas.

For schemata of predicate calculus, counterexamples are also referred to as *countermodels*. As we mentioned in §3.6.3, we can restrict ourselves to models in which every element in the domain has a name. We do this in examples (a)–(h).

(a) To begin with, a simple invalid argument schema: $\exists xLx/\forall xLx$ (the translation of a natural argument schema like *There are liars. So everyone is a liar*).

Proof: (that the schema is not valid). We need for this purpose a model **M** with $V_M(\exists xLx) = 1$ and $V_M(\forall xLx) = 0$. Any such model is called a counterexample to, or countermodel for, the schema. In this case it is not difficult to construct a counterexample. For example, let $D = \{1, 2\}$, $I(L) = \{1\}$, $I(a_1) = 1$, and $I(a_2) = 2$. Then we have $V_M(\exists xLx) = 1$, since $V_M(La_1) = 1$ because $1 \in I(L)$. And on the other hand, $V_M(\forall xLx) = 0$, since $V_M(La_2) = 0$, because $2 \notin I(L)$. A more concrete countermodel **M′** built on the same lines is this. We assume that Anne is a liar and that Betty is not. We take $D_{M'} = \{\text{Anne, Betty}\}$, $I_{M'}(L) = \{\text{Anne}\}$ and also $I_{M'}(a_1) = \text{Anne}$ and $I_{M'}(a_2) = \text{Betty}$. Then exactly the same reasoning as above shows that $V_{M'}(\exists xLx) = 1$, while $V_{M'}(\forall xLx) = 0$. It is even more realistic if **M″** is defined with $D_{M''}$ = the set of all people and $I_{M''}(L)$ = the set of all liars. If we once again assume that Anne is a liar and Betty is not and introduce a vast number of other constants in order to give everyone else a name too, then much the same reasoning as above again gives $V_{M''}(\exists xLx) = 1$ and $V_{M''}(\forall xLx) = 0$. It should be fairly clear not only that abstract models are easier to handle but also that they help us to avoid smuggling in presuppositions. In what follows, then, the counterexamples will all be abstract models with sets of numbers as their domains.

(b) Now for a very simple example of a valid argument schema: $\forall xSx/Sa_1$ (for example, as the translation of *Everyone is mortal. Thus, Socrates is mortal*). We have to show that $V_M(Sa_1) = 1$ for every suitable model **M** such that $V_M(\forall xSx) = 1$. Let us assume that. Then for every constant a interpreted in **M**, $V_M(Sa) = 1$. The constant a_1 must be interpreted in **M**, since **M** is suitable for $\forall xSx/Sa_1$. So it must be the case that $V_M(Sa_1) = 1$. We have now proved that $\forall xSx \models Sa_1$.

(c) The valid schema $\forall x(Mx \to Sx), Ma_1/Sa_1$ (a translation of *All men are mortal. Socrates is a man. Thus, Socrates is mortal,* for example) is slightly more complicated. Let **M** be suitable for this schema and $V_M(\forall x(Mx \to Sx)) = V_M(Ma_1) = 1$. Then $V_M(Ma \to Sa) = 1$ must hold for every constant a which is interpreted in **M**, so in particular we have $V_M(Ma_1 \to Sa_1) = 1$. Together with $V_M(Ma_1) = 1$, this directly implies that $V_M(Sa_1) = 1$. So we have now shown that $\forall x(Mx \to Sx), Ma_1 \models Sa_1$.

(d) The schema $\forall x\exists yLxy/\exists y\forall xLxy$ (a translation of *Everybody loves somebody. Thus, there is somebody whom everybody loves*) is invalid. In order to demonstrate this we need a model **M** in which L is interpreted and such that $V_M(\forall x\exists yLxy) = 1$ while $V_M(\exists y\forall xLxy) = 0$. We choose $D = \{1, 2\}$, $I(a_1) = 1$, and $I(a_2) = 2$ and $I(L) = \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$ (so we interpret L as the relation of inequality in D: the pairs $\langle 1, 1\rangle$ and $\langle 2, 2\rangle$ are absent in I(L)). Now we have $V_M(\forall x\exists yLxy) = 1$, because (i) $V_M(\exists yLa_1y) = 1$, since $V_M(La_1a_2) = 1$; and (ii) $V_M(\exists yLa_2y) = 1$, since $V_M(La_2a_1) = 1$. But on the other hand, we have $V_M(\exists y\forall xLxy) = 0$, because (iii) $V_M(\forall xLxa_1) = 0$, since $V_M(La_1a_1) = 0$; and (iv) $V_M(\forall xLxa_2) = 0$, since $V_M(La_2a_2) = 0$. So we have now shown that $\forall x\exists yLxy \not\models \exists y\forall xLxy$. Interpreting L as the relation of equality also gives a counterexample, and in view of the translation, this is perhaps more realistic. The counterexample given in (d) can easily be modified in such a way as to give a counterexample to the argument schema in (e).

(e) $\forall x(Ox \to \exists y(By \wedge Lxy))/\exists y(By \wedge \forall x(Ox \to Lxy))$ (a translation of *All logicians are reading a book. Thus, there is a book which all logicians are reading,* for example). The counterexample given in (d) will also work as a counterexample for this schema, if we take $I(O) = D$ and $I(B) = D$. Technically, this is quite correct, but nevertheless one might have objections. The informal schema of which this purports to be a translation seems to implicitly presuppose that logicians are not books, and books are not logicians, and that there are more things in our world than just logicians and books. These implicit presuppositions can be made explicit by including premises which express them in the argument schema. The schema thus developed, $\forall x(Ox \to \neg Bx), \exists x(\neg Ox \wedge \neg Bx), \forall x(Ox \to \exists y(By \wedge Lxy))/\exists y(By \wedge \forall x(Ox \to Lxy))$, is no more valid than the original one. In a countermodel **M′** we now choose $D_{M'} = \{1, 2, 3, 4, 5\}$, $I(a_1) = 1$, $I(a_2) = 2$, etc., $I(O)=\{1, 2\}$, $I(B) = \{3, 4\}$, and $I(L) = \{\langle 1, 3\rangle, \langle 2, 4\rangle\}$. Then it is not too difficult to check that we do indeed have $V_{M'}(\forall x(Ox \to \neg Bx)) = V_{M'}(\exists x(\neg Ox \wedge \neg Bx)) = V_{M'}(\forall x(Ox \to \exists y(By \wedge Lxy)) = 1$, while $V_{M'}(\exists y(By \wedge \forall x(Ox \to Lxy)))=0$.

(f) $\exists y\forall xLxy/\forall x\exists yLxy$ (a translation of *There is someone whom everyone loves. Thus everyone loves someone,* for example). Unlike the *quantifier switch* in (d), this quantifier switch is valid. Suppose $V_M(\exists y\forall xLxy) = 1$. We have to show that then $V_M(\forall x\exists yLxy) = 1$.

According to the assumption, there is a constant a interpreted in **M** such that $V_M(\forall xLxa) = 1$. This means that $V_M(Lba) = 1$ for every constant b which is interpreted in **M**. Now for any such b, it must also hold that $V_M(\exists yLby) = 1$, so that $V_M(\forall x\exists yLxy) = 1$ is guaranteed and $\exists y\forall xLxy \models \forall x\exists yLxy$ is proved. The proof that reversing (e) results in a valid argument schema is a little more complicated but goes along the same lines.

(g) $\forall xMx/\exists xMx$ (a translation of *Everyone is mortal. Thus, someone is mortal,* for example). Suppose **M** is suitable for this schema and that $V_M(\forall xMx) = 1$. Then we have $V_M(Ma) = 1$ for every constant a which is interpreted in **M**. There must be some such constant, since we have agreed that domains may never be empty, while in our approach A every element in the domain has a name. So $V_M(\exists xMx) = 1$. We have now proved that the schema is valid: $\forall xMx \models \exists xMx$. The validity of this schema depends on our choice of nonempty domains. In addition, Aristotle considered only predicates with nonempty extensions. So in his logic—unlike modern logic—the following schema was valid.

(h) $\forall x(Hx \to Mx)/\exists x(Hx \wedge Mx)$ (a translation of *All men are mortal. Thus, some men are mortal,* for example). As a counterexample we have, for example, **M** with $D_M = \{1\}$, $I(H) = I(M) = \varnothing$, and $I(a_1) = 1$. For then we have $V_M(Ha_1 \to Ma_1) = 1$, so that $V_M(\forall x(Hx \to Mx)) = 1$, while $V_M(Ha_1 \wedge Ma_1) = 0$, so that $V_M(\exists x(Hx \wedge Mx)) = 0$. If this seems a bit strange, then it should be remembered that this schema can also be seen as a translation of the intuitively invalid schema *All unicorns are quadrupeds. Thus, there are unicorns which are quadrupeds.* Furthermore, the original translation involves the implicit presupposition that there are in fact 'men', in the archaic sense of human beings. This presupposition can be made explicit by adding a premise which expresses it, and the resulting argument schema, $\forall x(Hx \to Mx), \exists xHx/\exists x(Hx \wedge Mx)$, is valid. In order to see this, let **M** be any model which is suitable for this schema and such that $V_M(\forall x(Hx \to Mx)) = 1$ and $V_M(\exists xHx) = 1$. We now have to show that $V_M(\exists x(Hx \wedge Mx)) = 1$. The second assumption gives us a constant a which is interpreted in **M** and for which $V_M(Ha) = 1$. From the assumption that $V_M(\forall x(Hx \to Mx)) = 1$ it follows that, in particular, $V_M(Ha \to Ma) = 1$, from which it follows with the truth table for $\to$ that $V_M(Ma) = 1$, and then with the truth table for $\wedge$ that $V_M(Ha \wedge Ma) = 1$. Now it follows directly that $V_M(\exists x(Hx \wedge Mx)) = 1$.

Exercise 2

Show that the argument schemata below are invalid by giving counterexamples.

(a) $\exists xAx, \exists xBx/\exists x(Ax \wedge Bx)$.
(b) $\forall x(Ax \vee Bx)/\forall xAx \vee \forall xBx$.
(c) $\forall x(Ax \to Bx), \exists xBx/\neg\exists xAx$.
(d) $\exists x(Ax \wedge Bx), \exists x(Bx \wedge Cx)/\exists x(Ax \wedge Cx)$.
(e) $\forall x(Ax \vee Bx), \exists x\neg Ax, \exists x\neg Bx, \forall x((Ax \wedge Bx) \to Cx)/\exists xCx$.
(f) $\neg\forall x(Ax \to Bx), \neg\forall xBx/\forall xAx$.

(g) $\forall xAx/\exists x(Bx \wedge \neg Bx)$.
(h) $\forall x\exists yRxy/\exists xRxx$.
(i) $\forall xRxx/\forall x\forall yRxy$.
(j) $\exists x\forall yRxy, \forall xRxx/\forall x\forall y(Rxy \vee Ryx)$.
(k) $\forall x\exists yRxy, \forall x(Rxx \leftrightarrow Ax)/\exists xAx$.
(l) $\forall x\exists yRxy, \forall x\forall y(Rxy \vee Ryx)/\forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)$.
(m) $\forall x\exists yRxy, \forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)/\exists xRxx$.
(n) $\forall x\forall y(Rxy \rightarrow Ryx), \forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)/\exists xRxx$.
(o) $\exists x\exists y\forall z(x = z \vee y = z)/\forall x\forall y(x = y)$.
(p) $\forall x\exists y(x \neq y)/\exists x\exists y\exists z(x \neq y \wedge x \neq z \wedge y \neq z)$.
(q) $\forall x\exists y(Rxy \wedge x \neq y), \forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)/\forall x\forall y(x = y \vee Rxy \vee Ryx)$.
(r) $\forall x(Ax \leftrightarrow \forall yRxy), \exists x\forall y(Ay \leftrightarrow x = y)/\forall x\forall y((Rxx \wedge Ryy) \rightarrow x = y)$.