I bring this section to a close with a brief remark on entailment and inconsistency. Now that we have defined what an interpretation is, and what truth in an interpretation is, there is no problem over defining these notions. A set of formulae Γ is inconsistent, i.e. $\Gamma \vDash$, iff (*a*) all the formulae in the set are closed (so that there are interpretations in which they have truth-values), and (*b*) there is no interpretation in which they are all true. A set of formulae Γ entails a formula ϕ, i.e. $\Gamma \vDash \phi$, iff (*a*) ϕ and all the formulae in Γ are closed, and (*b*) there is no interpretation in which all the formulae in Γ are true and the formula ϕ is false. As a special case of this, a formula ϕ is valid, i.e. $\vDash \phi$, iff (*a*) ϕ is closed, and (*b*) there is no interpretation in which ϕ is false. This is equivalent to saying: in every interpretation of the language of ϕ, ϕ is true. For if ϕ is closed, then in every interpretation of its language it must receive one, and only one, of the two truth-values.

On our second method of explaining what an interpretation is, it may seem reasonable to say that it is not *only* closed formulae that can be true. For truth was defined as satisfaction by all assignments, and this is a notion that applies to open formulae too. In fact this suggestion treats an open formula, standing alone, as identical with what is called its *universal closure*, i.e. the result of prefixing to it (in any order[19]) enough universal quantifiers to bind all its free variables. For the one will count as true (in a given interpretation) iff the other does. Now there would be no harm in extending the notion of truth in this way, so long as we take validity as our basic semantic notion, and either we do not talk of entailment and inconsistency at all, or we define them in terms of validity (as on p. 123). That is, it does no harm to count certain open formulae as valid, namely those whose universal closures are valid. But it can lead to a breakdown in expected relationships if we apply this idea to entailment or to inconsistency as these notions are ordinarily understood. For example, if the open formula *Fx* is true when and only

[19] Since we speak of *the* universal closure of a formula, we should strictly speaking specify some definite order, say alphabetical. But the order will make no difference to the truth-conditions of the formula.

when its universal closure $\forall xFx$ is true, then according to the usual definition of entailment it must hold that

$$Fx \vDash \forall xFx.$$

On the other hand it does *not* hold that

$$\vDash Fx \rightarrow \forall xFx.$$

For the universal closure of this formula is $\forall x(Fx \rightarrow \forall xFx)$, which is certainly not valid. Similarly with inconsistency. It will hold that

$$Fx, \neg\forall xFx \vDash$$

but not

$$Fx \wedge \neg\forall xFx \vDash.$$

These seem to me to be very paradoxical results. Some authors avoid them by revising the usual definitions of entailment and inconsistency so that these are now defined in terms of satisfaction rather than truth (e.g. Newton-Smith 1985: 193), but it is surely more straightforward to prevent the problem arising in the first place by insisting that it is only closed formulae that have truth-values. At any rate, that is the course that I shall take, and I shall not count $\vDash$ as defined in application to open formulae.