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Comments on Paul Hovda's 'Semantics as Information About Semantics Values'

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In SR(7-9), I posed the 'antimony of the variable'. How can the pair of variables x, y have a different semantic role from the pair x, x when x has the same semantic role as y? In attempting to solve this antimony, I suggested that we appeal to the idea of the values that are taken, not merely by a single variable, but by a sequence of variables (SR, 23-4). The semantic role of the two pairs of variables can then be distinguished, since the first pair will take a distinct pair of objects from the domain as values (assuming that the domain contains at least two objects) while the second pair will not.

Hovda's makes a marvelous alternative suggestion. 'The basic idea', he writes (pp. 4-5) 'is that a variable *can* refer to anything and *must* refer to exactly one thing.' The more usual idea is that a variable *actually* takes all of the objects in the domain as values (or 'referents'). My own relational account of variables is an instance of this approach, but with the modification that variables can now take their values *simultaneously* and not merely *singly*.

It has been quite common to think of quantification in modal terms but that is because the modal operators have themselves been regarded as a form of disguised quantification. The present proposal is quite different. For it is the *semantics* for the quantifier, not the quantifier itself, that now assumes a modal form. The quantifiers have often been treated as a paradigm of an extensional construction; and this makes it odd to think of them as having a modal semantics. But it is very natural, from an intuitive point of view, to think of a variable as taking the various objects from the domain as *possible* values; and once we make this idea precise, we end up with something like the present proposal.

However, the relevant notions of necessity and possibility must be properly understood. As Hovda points out, *metaphysical* necessity and possibility are not here in question (p. 7). We are not envisaging changes in the contingent circumstances under which a variable will now take this value and now take that value (no one, so to speak, *does anything* to make the values change). Rather, what is required is some form of *semantic* necessity and possibility, of the very kind that I had attempted to delineate in SR (43-44).[1] Thus given any variable and any object from the domain, it will be compatible with the variable meaning what it does (and, generally, with the language meaning what it does) that the variable should take the object as its value.

Hovda provides an outline of how he would like to develop this basic idea but there is one point, in his development, over which I have some misgiving. For he assumes that 'it is required [i.e. a semantic necessity] ... that the variable x refer to exactly one thing (in the domain of discourse)' (p. 6). Since the requirement operator is factive, it follows that the variable x actually refers to one thing. But what is this one thing? Hovda writes (p. 6), 'the "actual" value (referent) of the variable is immaterial; what matters are its possible values'. But the actual value is worse than immaterial - there is nothing it can sensibly be taken to be.

To avoid this awkwardness, it would be better, it seems to me, to weaken the requirement that the variable refer to exactly one thing and only require that it refer to at most one thing. One might also require that the variable refer to exactly one thing in any non-actual situation, though this is not strictly necessary for the semantics to go through.

Hovda develops the modal approach under 'the simplifying assumptions that we are giving the semantics with respect to a fixed domain of discourse D and fixed interpretations of the predicates' (p. 6). By 'fixed' here, he presumably means *modally* fixed; it is being assumed that the domain and the extension of the predicates will not vary from 'world to world'. Without these assumptions, the obvious clause for the existential quantifier will no longer be correct, as he is no doubt aware. For consider the special case of an existential formula ∃vFv with no free variables. We will want to lay down as a requirement that:

> ∃vFv is satisfied iff it is possible that for some x, Fv is satisfied and v refers to x.

But the right hand side may be true for the wrong reason. For even though nothing in fact F's it may be semantically possible that

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[1] Though I should point out that the distinction between semantic fact and requirement (SR, 49-50) is not so relevant in the present context.

something F's and hence semantically possible that for some x, Fv is satisfied and v refers to x.

If we are to do without these fixity assumptions, we will need to modify the requirement on the existential formula to something like:

∃vFv is satisfied iff there is a set *D* which is the domain and a set *F* which is the extension of F, for which it is possible that for some x in *D*, Fv is satisfied, v refers to x and the extension of F is *F*.

Thus just as the pre-existing values of the free variables must be carried over to the realizing possibility in Hovda's original clause for the existential quantifier, so must the pre-existing domain and the pre-existing extensions of the predicates. The 'existence principle' for the assignment of values to variables must also be strengthened. It is no longer sufficient to require that, for any assignment, it should be possible that each variable refers to its value under that assignment; we should also require that this be possible when the domain and extensions of the predicates remain the same.

The strengthened assumption is a kind of conservativity principle; the semantic facts can hold (in this case they take the form of an assignment of values to the variables) no matter what the non-semantic facts. I had remarked on the conservativity of semantics and of the analogy with the conservativity of mathematics in SR, 128-9. What I find remarkable here is that conservativity is not merely a requirement *of* the semantics but a requirement *within* the semantics; the semantics will not deliver the required results unless the assumption of conservativity is explicitly made.

The modal semantics has a number of advantages over my own relational semantics. First, it is somewhat more economical in its choice of semantic primitives. As Hovda points out, it makes no use of 'a distinctively relational apparatus of semantic values'. It does adopt as primitive the notion of a variable *singly* taking a value, but not the notion of some variables (or open expressions) *simultaneously* taking some values. I might add that, in contrast to Tarski's approach, the modal semantics makes use of a non-relational notion of satisfaction (something more akin to truth). We do not talk of an assignment *satisfying* a formula but of a formula being *satisfied*; and instead of having an explicit specification of an assignment, we can suppose that the assignment is implicitly given through the 'world' of evaluation. The modal semantics does make use of the notion of a semantic requirement, in contrast to the relational approach, but this is no great loss, since the notion will be required, in any case, for other purposes.

Second, the modal semantics provides us with a more unified account of the semantics for variables and names. Indeed, Hovda's modal semantics for variables can be seen to be modelled on the modal semantics that I, in effect, provided for names - but with the fixed reference relation for names (for which it is semantically required that if a name refers to x then it is semantically required that the name refers to x) replaced by a variable reference relation. The relational connections between variables or between names are then delivered in each case by the modal connections. Thus just as it will not be a semantic requirement that 'Cicero' corefer with 'Tully', so it will not be a semantic requirement that 'x' corefer with 'y'.

Third, and perhaps most important of all, the modal semantics may plausibly be regarded as more fundamental than the relational semantics. For suppose we ask: what is it for some variables (or open expressions) simultaneously to take certain values? Then a plausible answer is that it is a matter of the variables being *capable* of referring to those values (with the non-linguistic facts remaining the same). Thus it appears that the distinctively relational primitive of the relational semantics can be defined in terms of the more basic primitives of the modal semantics.

But this is not to say that the relational semantics for variables (and the corresponding coordinative semantics for names) is of no independent interest. For there are various things we may want of a semantics. One is fundamentality—we wish to state the semantics in the most fundamental or 'rock bottom' terms. Another is perspicuity—we wish to make clear certain features of the semantics or how it is being capable of being put to certain uses. But, as has commonly been observed, these two desiderata are often in tension with one other and we may therefore need to sacrifice a certain degree of fundamentality if we wish to achieve a certain level of perspicuity.

There are at least three respects, I believe, in which the relational semantics for variables and the corresponding coordinative semantics for names are more perspicuous than their modal counterparts. First, they are extensional. They employ notions that should be acceptable to someone who was only willing to accept extensional notions. Indeed, the notion of the values simultaneously taken by some variables or open expressions is one that can already be defined within the standard extensional semantics for first order logic. Thus there is a sense in which our own semantics for first order logic does not go beyond the conceptual resources of the standard semantics. The case of names is more problematic, but even here it is not clear that the sceptic about semantic necessity need be uncomfortable with the notion of coordination.

Second, these semantical accounts are proto-mathematical. We know how to convert each of them into a piece of mathematics, to which the typical methods of model theory can then be applied. We are able, in particular, to define clear mathematical notions of validity and consequence about which various results, such as soundness and completeness, can then be proved. But the mathematical content of the modal semantics is not so clear. How are validity and consequence to be defined? In modal terms? But then how are the modal terms to be incorporated into the mathematics and the proof of mathematical results? I do not want to say that these problems cannot be solved; it is merely that the use of modal notions stands in the way of our seeing what the underlying mathematical structure should be.

Third, these semantical accounts are compositional. They tell us what the semantic values or connections should be and how they are to be compositionally determined. What is of particular importance is that we are thereby able to see that the semantic values or connections are not inappropriately linguistic or arbitrary. One of my major criticisms of the Tarski semantics was that it made the semantic values of open expressions inappropriately linguistic (SR, 11). For the semantic values will themselves be constituted by assignments, i.e. functions from *variables* to objects; and nor, might I add, does it help to replace assignments by *sequences* of objects, since the semantic values will then arbitrarily impose a given order on the variables. One of the major motivations for my developing the relational semantics was to show how the demand of compositionality might be met, with non-linguistic and non-arbitrary semantic values or connections in place of the linguistic or arbitrary semantic values of Tarski.

Hovda claims that his modal semantics is compositional (p. 7). But the sense in which this is so is not clear; and it is certainly not clear that the demand of compositionality described above can be met. For what are these non-linguistic and non-arbitrary semantics values with respect to which the semantics is compositional? There is nothing in the statement of his requirements themselves that enables us to say; and we can see the relational semantics as telling us what it is about the modal semantics (or what it is in general) that enables the demand for compositionality to be met. The remarkable lesson we learn, not evident from the modal semantics itself, is that this demand can only be met by adopting a relational conception of semantic value.

It is not clear to me how 'down' Hovda wishes to be on the more explicitly relational form of the semantics. He cannot wish to dismiss it altogether since he appears to grant that the relational form of the semantics is reducible (or, at least, largely reducible) to the modal semantics (p. 1). But he does wish to confine it to the 'periphery' and he does suggest that it is misleading or inadequate in various respects.

Perhaps I can agree.[2] But I do not think that his remark that 'the periphery might result from an attempt to force the new model into the old mold' is warranted. The motivation for the semantics arises from a very real problem; and it would appear to be necessary, if this problem is to be solved, that we take explicit note of the semantical relationships among different expressions.

Let me conclude by discussing a couple of more technical topics raised by Hovda's contribution. In SR (sections IIB, D), I suggested that the operators for semantic necessity or for the propositional attitudes might more plausibly be taken to be closed under 'manifest' consequence rather than under classical consequence. The logic **QK** of an operator when it is taken to be closed under classical consequence, is familiar; and using this logic as a model, Hovda shows how one may construct a logic **M** for an operator when it is merely taken to be closed under manifest consequence (pp. 2-3). The basic idea is to use a syntactic correlate of manifest consequence in order to state the closure rule.

In a footnote (p. 3), Hovda also sketches a semantics for **M** in terms of structured propositions and propositional counterparts. This semantics is rather different from the standard semantics for **QK** (which makes no appeal to structured propositions but simply evaluates formulas at worlds); and so it is worth pointing out that it is possible to give a semantics **M** that is much closer to the standard semantics for **QK**.

A standard model for **QK** is of the form (W, D, R, v), where W is a non-empty set of worlds, D is function taking each world into a non-empty set of individuals, R is the accessibility relation on worlds, and v is a valuation telling us how the predicates will apply to the individuals of each world. Hovda assumes 'increasing domains':

if wRv then $D_w \subseteq D_v$,

though this assumption could easily be dropped under a suitable modification of the logic.

We may obtain a similar semantics for the logic **M** by taking a model to be of the form (W, D, G, γ, R, v). W, D, R and v are as before. G is now a function taking each world w into a set of 'guises' G_w and, for each world w, γ_w is a function taking each guise g of G_w

[2] Though I would wish to demur from his doubts as to 'how well this semantics solves the "antimony of the variable"' (p. 5). For even if we allow for the possibilities of coordination among the variables, the pair of variables x, x will still differ semantically from the pair of variables x, y in that the former pair will be ambiguous between an interpretation in which the values of the variables must be the same and an interpretation in which they may be different while the latter pair will only be capable of having an interpretation in which the values may be different. In the identity formula 'x = x', only the first of these interpretations of the pair x, x will be acceptable.

into the individual $\gamma_w(g)$ of D_w of which it is a guise (it should be assumed that the function is onto). In place of increasing domain, we assume:

if wRv then $G_w \subseteq G_v$.

Thus the accessible 'alternatives' to w will concern not the individuals of w but the guises under which those individuals have been conceived.

We now wish to evaluate $\Box\varphi(v_1, v_2, ..., v_n)$ at a world w, where v_1, v_2, ..., v_n are taken to be *occurrences* of free variables in $\varphi(v_1, v_2, ..., v_n)$, not simply the free variables. Suppose that the individuals a_1, a_2, ..., a_n have been assigned to v_1, v_2, ..., v_n (so a_i must be the same as a_j when v_i and v_j are occurrences of the same variable). The clause is then as follows:

w $|=$ $\Box\varphi[a_1, a_2, ..., a_n]$ iff for some guises g_1, g_2, ..., g_n of G_w, $\gamma_w(g_1) = a_1$, $\gamma_w(g_2) = a_2$, ..., $\gamma_w(g_n) = a_n$, and v $|=$ $\varphi[g_1, g_2, ..., g_n]$ whenever wRv.

Thus the clause is like the standard clause, but mediated through the guises by which the individuals in question might have been conceived. Of course, for a relationist like myself, these guises are not to be taken in a Fregean spirit as modes of presentation but as the abstract counterpart of coordinative chains.

The other topic concerns the more radical logical treatment of propositional attitudes considered in section IV.G of SR. I there suggested that an existential formula such as ∃x(Bel[P(x)] & Bel[Q(x)]) might be evaluated as follows. We first consider, in the standard way, whether some individual satisfies Bel[P(x)] & Bel[Q(x)]. But the question of whether a given individual satisfies Bel[P(x)] & Bel[Q(x)] will not now turn, as it would under the classical semantics, on whether it satisfies each of Bel[P(x)] and Bel[Q(x)]. Rather, it will turn on whether it simultaneously satisfies Bel[P(x)] and Bel[Q(x)], which is a matter of whether the agent has a coordinated belief that the object P's and that the object Q's. Thus there will be a difference between whether an individual *a* satisfies Bel[P(x)], Bel[Q(x)] and whether an identical pair of individuals *a*, *a* satisfies Bel[P(x)], Bel[Q(y)], with only the former requiring a coordinated belief.

The semantics is clearly relational, since the base clauses for the belief operator will not concern the satisfaction of individual belief reports but the satisfaction of a composite belief report. Hovda wonders whether one might somehow 'reduce the appeal to coordinated formulas and content' in favor of a modal operator (p. 9). It is not altogether clear to me how his own suggestion can be made to work, for in saying something like ∃x□(Bel[P(x)] & Bel[Q(x)]), where □ represents "the communal body of information", we would appear to be saying something stronger than what is required. An alternative suggestion might go as follows. Let us use Blf(b) to indicate that b is a belief (of the given agent), b:φ to indicate that b is a belief that φ, and $\Box\psi$ to indicate that it is representational requirement that ψ. Then we might understand $\exists$x(Bel[P(x)] & Bel[Q(x)]), under the preferred reading, to mean: $\exists$b$\exists$c(Blf(b) & Blf(c) & $\Box\exists$x(b:Px & c:Qx)), where $\Box$(b:Px) & $\Box$(c:Qx) should not be taken to imply $\Box\exists$x(b:Px & c:Qx), given the manifest character of the operator $\Box$.