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Semantics as Information about Semantic Values

PAUL HOVDA
Reed College

I suggest that the core ideas of Kit Fine's *Semantic Relationism* are the notion of *semantic requirement* and the notion of *manifest consequence*, the non-classical logical relation associated with *semantic requirement*. Surrounding this core are novel "relational" systems of *coordinated sequences of expressions*, *relational* (as opposed to intrinsic) *semantic values*, *coordinated propositions*, and *coordinated content*. I take Fine to take the periphery to be reducible to the core (but see below). I will make some primarily exegetical remarks about the two core ideas, and then make more critical remarks about the periphery. I should say that I find the book, as a whole, illuminating and, for the most part, convincing. I hesitantly suggest that the core constitutes an important and novel model for thinking about semantics (and representation in general), while the periphery might result from an attempt to force the new model into the old mold.

Requirements, Information, and Manifest Consequence

Fine presents the notions of semantic requirement and manifest consequence in Chapter Two, especially in Sections B through D. The crucial idea is that what is semantically required is not closed under classical logical consequence, but under a weaker kind of consequence. Manifest consequence can be thought of as the kind of consequence that is available to an ideal cognizer *regardless of her "takes" on the various objects of her knowledge*. So it is not a manifest consequence of the two premises that Superman flies and that Superman works in the office that Superman both flies and works in the office, because one might know (as Lois Lane does) the two premises *via* two different "takes" on Superman.

Since semantics (the body of facts that are semantically required) is closed only under manifest consequence, we get the result that it can be semantically required that ‘Hesperus’ refer to Hesperus, required that ‘Phosphorus’ refer to Hesperus, and not required that ‘Hesperus’ and ‘Phosphorus’ refer to the same thing.

For concreteness, allow me to present a sketch of a formalization for the closure of semantic requirement under manifest consequence that provides an easy comparison to a more familiar modal logic.

First, the familiar logic: given a first-order modal language $\mathcal{L}(\Box)$, we can get the consequence relation $\vdash_{\mathbf{QK}}$ for the quantified modal logic **QK** (quantified **K**) that is sound and complete for the normal increasing-domain Kripke semantics with no constraints on the accessibility relation, by simply extending any derivation system that generates the standard[1] first-order $\vdash$ so as to have its rules govern wffs of $\mathcal{L}(\Box)$ as well and to incorporate the following rule. If Γ is any set of wffs, $\Box(\Gamma)$ is $\{\Box\gamma{:}\gamma \in \Gamma\}$; the rule is this:

For any set of wffs Γ, and any wff B, if

$$\Gamma \vdash_{\mathbf{QK}} B$$

then

$$(\Gamma) \vdash_{\mathbf{QK}} B.$$

We allow the limit case in which Γ is empty.[2]

The rule basically says that “$\Box$ is closed under consequence.” Thus the modal logic **QK** is the result of adding, to classical first-order logic, a one-place sentential operator with the sole constraint that it be “closed” in this way.

We use ‘\$’ to represent “it is required that.” The logic **M** that we will describe results from adding to classical first-order logic the constraint that \$ be closed under (one way of formalizing) *manifest* consequence.[3] Using language from p. 49 of *SR*, we might say that **QK** gives the bare logic for domains of propositions taken factually, reading $\Box$ as something like “domain d includes the fact that” while

1 I.e., in which formulas with free variables may occur in premises and are not given the “generality interpretation,” but are interpreted as “parameters.”

2 In this case, our rule reduces to the Necessitation rule of more familiar presentations. An instance of the familiar **K** axiom, e.g., $\Box(Fx \to Gx) \to (\Box Fx \to \Box Gx)$ is obtained by observing that $(Fx \to Gx), Fx \vdash_{\mathbf{QK}} Gx$ and hence, applying our rule, $\Box(Fx \to Gx), \Box Fx \vdash_{\mathbf{QK}} \Box Gx$, and twice applying Conditional Proof. Every Converse Barcan Formula, e.g., $\Box\forall x Fx \to \forall x \Box Fx$, is a theorem. Since $\forall x Fx \vdash_{\mathbf{QK}} Fy$ (by Universal Instantiation), $\Box\forall x Fx \vdash_{\mathbf{QK}} \Box Fy$ (applying the rule), and hence $\Box\forall x Fx \vdash_{\mathbf{QK}} \forall x \Box Fx$ (by Universal Generalization).

3 In footnote 11 of Chapter Two, Fine mentions a couple of variations on the notion in the main text; after playing with a couple of them, this seemed to me to be the most natural.

M gives the logic for domains taken as information, reading $\$$ as "domain *d* includes the information that."

Let Γ be any set of wffs of $\mathcal{L}(\leftrightarrow)$. Say that Γ is *differentiated* if for each variable in the language, it occurs free in at most one $\gamma \in \Gamma$, and occurs free at most once within any $\gamma \in \Gamma$. Let $\$(\Gamma)$ be $\{\$\gamma{:}\gamma \in \Gamma\}$. The distinctive rule for manifest consequence is this:

For any differentiated set of wffs Γ, and any wff B, if

$$\Gamma \vdash_{\mathbf{M}} B$$

then

$$\$(\Gamma) \vdash_{\mathbf{M}} \$B.$$

The requirement of differentiation corresponds to the idea that to draw the *manifest* consequences of some premises, one allows oneself to see *each isolated occurrence* of each object that occurs in the premises, but simultaneously blinds oneself to the *recurrences* of an object among the premises. The consistency of **M** follows immediately from the consistency of **QK**, since, obviously, every application of the **M**-rule is an application of the **QK**-rule. Giving an intuitively satisfying formal semantics, roughly like the possible worlds semantics for **QK**, for which this system is sound and complete seems to be possible.[4] But with or without such a formal semantics, the logic has a clear motivation, and there is no need for a reductive account of the notion of requirement.

Some examples:

(e1a) $\$Fx, \$\neg Fx \nvdash_{\mathbf{M}} \$\exists y(Fy \wedge \neg Fy)$

(e1b) $\$Fx, \$\neg Fx \vdash_{\mathbf{M}} \exists y\$(Fy \wedge \neg Fy)$

(e2) $x = w \wedge \$Ryx \vdash_{\mathbf{M}} \$Ryw$

(e3) $\exists x\$(Ryx \wedge Rzx) \nvdash_{\mathbf{M}} \$\exists x(Ryx \wedge Rzx)$

(e4a) $\$\forall w(Rxw \leftrightarrow Ryw), \$Rxz \not\vdash_{\mathbf{M}} \$Ryz$

(e4b) $\$\forall v(v = x \rightarrow (\forall w(Rvw \leftrightarrow Ryw) \wedge Rvz)) \vdash_{\mathbf{M}} \$Ryz$

4 My own approach to this involves (i) regarding the wffs as naming structured "propositions" that are basically isomorphic to the wffs they name, (ii) an accessibility relation that relates worlds to sets of worlds, and (iii) for each set accessible from a given world, a well-behaved counterpart function that takes a proposition to a set of counterparts of it (so that occurrences of a single object might be replaced with multiple objects). In brief, $\$\phi$ is true at w if, for any set of worlds w can see, there is a counterpart of the proposition named by ϕ at w which is true at every world in the set. I make no use of "coordinated propositions."

(e1a) and (e1b) correspond to the situation with Lois Lane's believing that x (Superman, Clark Kent) flies and that x does not fly. The two pieces of information (Fx, $\neg Fx$) involve the same object, but it does not follow that the body of information in question includes that there be one object with both properties; yet the situation is properly described as one in which, of a single object, it is required (believed) that it have each of two contradicting properties.[5] (e2) emphasizes the validity of classical Substitutivity of Identicals. (e3) corresponds to the situation with the reference of 'Hesperus' and 'Phosphorus'; though there is something x (Venus) such that it is required, of each name, that it refer to x, it is not required that the two names co-refer. (e4a) and (e4b) show a complication that arises in trying to describe the situation with 'Phosphorus' (x) and '$\Phi\omega\sigma\phi o\rho o\varsigma$' ($y$), assuming these to be (unlike 'Hesperus' and 'Phosphorus') "strictly co-referential." The first premise of (e4a) corresponds to the requirement that the two names co-refer, and the second to the requirement that 'Phosphorus' refers to Venus. The expected conclusion does not follow because, in effect, the ideal cognizer may have different "takes" on the name 'Phosphorus', in the two premises; the situation could be like that "with Superman and Clark Kent." Thus, something like the premise of (e4b) is needed to insure that the information is properly "coordinated."

The issue raised by (e4a) and (e4b) invites the idea that we consider a special class of "transparent designators." We could introduce a new syntactic category of term, marked with a superscript *.[6] Then we supplement the logic to say that a premise set counts as differentiated even if it contains more than one occurrence of a transparent designator. Then, putting a^* and b^* for the two quotation-names, we would have

(e4c) $\$\forall w(Ra^*w \leftrightarrow Rb^*w), \$Ra^*z \vdash_{\mathbf{M}} \$Rb^*z$

The transparency of quotation-names is compatible with our using them to express the distinction between "accidental" and "strict" co-reference. It should also be noted that, if we treat them in a Russellian manner, "purely qualitative" definite descriptions are, effectively, transparent designators, for their occurrences in premises will be unabbreviated in such a way as to introduce no free occurrences of variables.

The Periphery: Variables

In Section G of Chapter One, Fine presents a relational semantics for first-order logic, on which semantic values are assigned to *coordinated*

5 To derive (e1b): derive as a theorem $\forall x \forall y(\$Fx \wedge \$\neg Fy \rightarrow \$(Fx \wedge \neg Fy))$.

6 For a person's beliefs as a body of information, it might be appropriate to include a single transparent designator corresponding to the first-person pronoun.

sequences (c-sequences) of expressions. A c-sequence is a sequence of expressions together with a coordination scheme for the expressions in it; for example, there are two c-sequences whose expression sequence is (x, x): one (σ^+) on which the two occurrences of x are coordinated, the other (σ^-) on which they are not (or are "negatively coordinated"). The relational semantics (RS) is compositional in that the semantic value of a c-sequence is a function of the values of its components, at least if we regard the coordination scheme as part of the syntax, as seems to be suggested on p. 30: "the syntactic object of evaluation will no longer be a sequence of expressions but a *coordinated* sequence of expressions." The differing values of σ^+ and σ^- can be computed from the values, of x and of x, together with the "value" or role of the coordination scheme.

It is not clear to me how well this semantics solves the "antinomy of the variable" from Section A (how can the pair of variables x,x differ semantically from the pair x,y, while the single variables x and y are semantically the same?). The negatively coordinated c-sequence whose expressions are (x, x) has the same semantic value as the negatively coordinated c-sequence whose expressions are (x, y); hence "that particular pair" of variables x, x is not semantically different from that pair x, y. Now, the mere expression sequence (x, x) does differ from the expression sequence (x, y) in that the first, but not the second, can be mated with a coordination scheme on which the two occurrences of expressions are coordinated. But it is not clear that this is a *semantic* difference between the mere expression sequences, rather than a syntactic one.

Thus, Section G presents a new system of semantic values, compositional with respect to a new system of syntax. A different perspective on the semantics of variables emerges if we recast a more familiar syntax and semantics for first-order logic in "modal" fashion, being explicit that semantics constitutes a body of information for which an **S5**-type extension of **M** is appropriate.[7] The basic idea about the semantics of variables is that a variable *can* refer to anything, and *must* refer to exactly one thing.[8] A little more formally:

7 In fact, what is wanted here is closer to **QK** plus the **S5** principles—as applied to semantical vocabulary and the auxilliary vocabulary of set theory. We get the needed effect within **M** if references to expressions are "transparent," which can be achieved either by the brute force method of introducing transparent names or by regarding the language for which the semantics is given as itself a set-theoretic (or other purely qualitative) construct.

8 One might not want to use the word "refer" here, but, instead, something like "has as value."

For each thing b (in the domain of discourse) it is allowed (not required that it not be so) ($\neg\$\neg$) that x refer to b and it is required ($) that the variable x refer to exactly one thing (in the domain of discourse).

The "actual" value (referent) of a variable is immaterial; what matters are its possible values. Similarly, what matters semantically for a wff with free variables is whether it *can* be satisfied.

Instead of appealing to σ^{+} in our semantics for $x=x$, we appeal to:

$ (any wff formed by flanking '=' with two variables v_1, v_2 is satisfied iff the referent of v_1 is identical with the referent of v_2).

(Assuming that quotation names are transparent designators) we will be able to deduce

$ ('$x=x$' is satisfied) and $\neg$$ ('$x=y$' is satisfied).

For we will also have enough in our semantics to show that

$ ('x' and 'x' refer to the same thing) and $\neg$$ ('x' and 'y' refer to the same thing),

which I take to express a semantic difference between the pair x, x and the pair x, y. I sketch a few more propositions of the semantics, with the simplifying assumptions that we are giving the semantics with respect to a fixed domain of discourse D and fixed interpretations of the predicates.

$ (For every variable v, v refers to exactly one object in D).

We assume that $\in$ *is "stable" and set theory is available with respect to* $: *e.g., we have that* $(for all x and y, $x \in y$ iff $$x \in y$). *Let an "assignment" be any function whose domain is the set of variables and whose range is D.*

Therefore: $ (there is exactly one assignment a such that for every variable v and every object $j \in D$ ($a(v)=j$ iff v refers to j)).

$ (for any assignment a: $\neg\$\neg$ (for every variable v, v refers to $a(v)$)).

$ (for any wff B of the form '$\exists v\phi$', B is satisfied iff there is some object $j \in D$ such that, if a is the function that characterizes the reference of the variables, and a' is the function just like it except that $a'(v)=j$, then ¬$¬ ($a'$ is the function that characterizes the reference of the variables, and ϕ is satisfied)).

Thus the standard Tarski-style semantics can be re-cast in "modal" fashion to yield a semantics that, we might say, assigns no particular values to variables and open wffs, yet assigns *possible* values in a compositional manner. (But note that this is not meant to be the "metaphysical" "could have won the election" sort of possibility.) The antinomy of the variable is resolved not by finding a value for the sequence (x, y) that is different from the value of (x, x) but merely by attending to some of the points Fine makes in Section F of Chapter One (and taking them to be constitutive of the semantics). This allows us to take what we might call a "relational" *perspective* on the familiar semantics, without a need for a distinctively relational apparatus of semantic values.

The periphery: Names and Coordinated Content

I make similar remarks about the "coordinated content" of Section F of Chapter Two: a relational perspective does not require a new apparatus. Fine's treatment of names in the earlier Sections of this Chapter calls for no novel semantic values. Roughly, what is provided is a new perspective on the referentialist semantic values, not an alternative system of values. "Cicero = Cicero" and "Cicero = Tully" are semantically different because the one, and not the other, is semantically required to be true. This holds even if the semantics is the familiar referentialist semantics, on which the semantic values of the sentences are in fact (but not by requirement!) the very same singular proposition. Semantics is informational, not factual, and this solves Frege's puzzle. "Sense" becomes *information* about reference, rather than a parallel compositional apparatus of values.

There is no need to introduce two distinct "coordinated propositions" for the two sentences to express. Besides being technically complicated, the novel system of coordinated content easily can be misunderstood to require a corresponding novel syntax, in which sentences are suspended in a web of invisible vincula.

Fine seems to be willing to reduce any use of the apparatus of coordinated content to the use of the idea of semantic requirement. He says, on p. 59,

> In saying that "Cicero = Cicero" expresses the positively coordinated proposition that $c=c$, what I am saying is that it is a semantic

> requirement that the sentence signifies an identity proposition whose subject and object positions are both occupied by the object *c* while, in saying that "Cicero = Tully" expresses the uncoordinated proposition that $c{=}c$, I am merely saying that it is a semantic requirement that it signifies an identity proposition whose subject position is occupied by *c* and whose object position is occupied by *c*.

But for some later uses of coordinated content, it is not entirely clear to me how the reduction would go.

When discussing the advantages of the relationist over the referentialist views in Section D of Chapter Three, Fine suggests that the correct explanation of the difference between the cognitive impacts of being told "Cicero is an orator" and being told "Tully is an orator" (supposing a certain information base) is that the one singular proposition (that both sentences express) becomes coordinated in a different way (with the base) as a result of the telling. One sees how this is supposed to go, and how the total coordinated content that would result from the one telling is different from the one that would result from the other. But it is hard to see *why* someone would coordinate in the one way rather than the other without "going linguistic" in something like the way Fine has suggested the referentialist might have to; this is perhaps more clearly so if we consider how we would reduce any appeal to coordinated content in the story. (And if the relationist must "go linguistic," at some stage of explanation, then Fine's criticism of the referentialist for "going linguistic" may lose some of its force.) And, finally, one need not appeal to coordinated content to characterize the difference between the resulting information sets; one set will include, in its closure under manifest consequence, stronger (non-coordinated) propositions than the other.

In Section G of Chapter Four, Fine presents a problem:

> We would like to report Peter as believing (or realizing) of some famous Pole that he is a pianist but not as believing (or realizing) both that he is a pianist and that he is a statesman. In symbols:
>
> $\exists x[Fx \wedge Bel[P(x)] \wedge \neg(Bel[P(x)] \wedge Bel[S(x)])]$
>
> [Similarly, in symbols]:
>
> $\exists x[Fx \wedge Bel[S(x)] \wedge \neg(Bel[S(x)] \wedge Bel[P(x)])]$
>
> [From these it follows, classically, that] there are two famous Poles

$$\exists x \exists y (x \neq y \wedge F(x) \wedge F(y))$$

which is clearly not our intention.

Fine suggests we accept 1 and 2 and give up the application of classical logic to them. The justification involves considering the *coordinated* formulas that are the matrix clauses of 1 and 2 (i.e., $Fx \wedge Bel[P(x)] \wedge \neg(Bel[P(x)] \wedge Bel[S(x)])$ and its counterpart from 2). Fine suggests that each formula is true of the object Paderewski (hence 1 and 2 are both true) yet the unwanted conclusion does not follow, because of the relational treatment of the coordinated formulas, so that "at the deepest semantical level, [the *Bel* operator] picks out a coordinated body of opinion, rather than an uncoordinated range of individual opinions …"

It is unclear to me just how to reduce the appeal to coordinated formulas and content. One might suggest that we see 1 and 2 as each involving a hidden informational operator $ tied to something like "the communal body of information" (appealing to a notion of community like that discussed at the end of Section F of Chapter Four). We might get

1′ $\exists x \$[Fx \wedge Bel[P(x)] \wedge \neg(Bel[P(x)] \wedge Bel[S(x)])]$

2′ $\exists x \$[Fx \wedge Bel[S(x)] \wedge \neg(Bel[S(x)] \wedge Bel[P(x)])]$

3′ $\$\exists x \exists y (x \neq y \wedge F(x) \wedge F(y))$

3′ does not follow in **M** from 1′ and 2′ (even though classical logic is valid in **M**). It is not clear to me, however, how similarly to get the result Fine aims for in footnote 11 of Chapter Four.