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*Reason and the Grain of Belief**

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1. Preview

This paper is meant to be four things at once: an introduction to a Puzzle about rational belief, a sketch of the major reactions to that Puzzle, a reminder that those reactions run contrary to everyday life, and a defence of the view that no such heresy is obliged. In the end, a Lockean position will be defended on which two things are true: the epistemology of binary belief falls out of the epistemology of confidence; yet norms for binary belief do not always derive from more fundamental ones for confidence. The trick will be showing how this last claim can be true even though binary belief and its norms grow fully from confidence and its norms.

The paper unfolds as follows: §2 explains Puzzle-generating aspects of rational belief and how they lead to conflict; §3 sketches major reactions to that conflict; §4 shows how they depart radically from common-sense; §5 lays out my solution to the Puzzle; §6 defends it from a worry about rational conflict; §7 defends it from a worry about pointlessness.

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2. The Puzzle

The Puzzle which prompts our inquiry springs from three broad aspects of rational thought. The first of them turns on the fact that belief can seem coarse-grained. It can look like a three-part affair: either given to a claim, given to its negation, or withheld. In this sense of belief we are all theists, atheists or agnostics, since we all believe, reject or suspend judgement in God. The first piece of our Puzzle turns on the fact that belief can seem coarse in this way.

This fact brings with it another, for belief and evidential norms go hand in hand; and so it is with coarse belief. It can be more or less reasonably held, more or less reasonably formed. There are rules (or norms) for how it should go; and while there is debate about what they say, exactly, two thoughts look initially plausible. The first is

> *The conjunction rule.* If one rationally believes P, and rationally believes Q, one should also believe their conjunction: (P&Q).

This rule says there is something wrong in rationally believing each in a pair of claims yet withholding belief in their conjunction. It is widely held as a correct idealisation in the epistemology of coarse belief. And so is

> *The entailment rule.* If one rationally believes P, and P entails Q, one should also believe Q.

This principle says there is something wrong with failing to believe the consequences of one's rational beliefs. It too is widely held as a correct idealisation in the epistemology of coarse belief. According to these principles, rational coarse belief is preserved by conjunction and entailment. The Coarse View accepts that by definition and is thereby the first piece of our Puzzle.

The second springs from the fact that belief can seem fine-grained. It can look as if one invests levels of confidence rather than all-or-nothing belief. In this sense of belief one does not simply believe, disbelieve or suspend judgement. One believes to a certain degree, invests confidence which can vary across quite a range. When belief presents itself thus we make fine distinctions between coarse believers. "How strong is your faith?" can be apposite among theists; and that shows we distinguish coarse believers by degree of belief. The second piece of our Puzzle turns on belief seeming fine in this way.

This too brings with it evidential norms, for degree of belief can be more or less reasonably invested, more or less reasonably formed. There are rules (or norms) for how it should go; and while there is debate about what they say, exactly, two thoughts look initially plausible. The first is

The partition rule. If P_1-P_n form a logical partition, and one's credence in them is cr_1-cr_n respectively, then $(cr_1 + \ldots + cr_n)$ should equal 100%.[1]

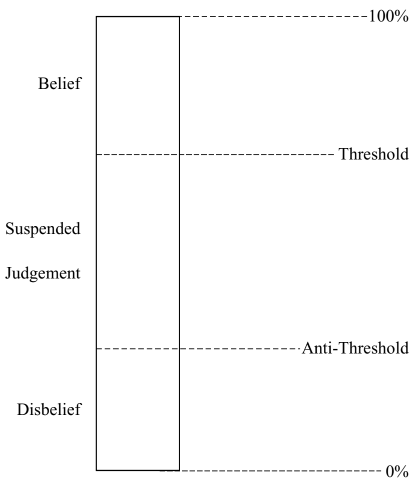
This rule says there is something wrong with investing credence in a way which does not sum to certainty across a partition. It is widely held as a correct idealisation in the epistemology of fine belief. And so is

The tautology rule. If T is a tautology, then one should invest 100% credence in T.

This rule says there is something wrong in withholding credence from a tautology. It too is widely held as a correct idealisation in the epistemology of fine belief. According to these principles: rational credence spreads fully across partitions and lands wholly on tautologies. The Fine View accepts that by definition and is thereby the second piece of our Puzzle.

The third springs from the fact that coarse belief seems to *grow* from its fine cousin. Whether one believes, disbelieves or suspends judgement seems fixed by one's confidence; and whether coarse belief is rational seems fixed by the sensibility of one's confidence. On this view, one manages to have coarse belief by investing confidence; and one manages to have rational coarse belief by investing sensible confidence. The picture looks thus:

A



The Threshold View accepts this picture by definition and is thereby the third piece of our Puzzle.

Two points about it should be flagged straightaway. First, the belief-making threshold is both vague and contextually variable. Our chunking of confidence into a three-fold scheme—belief, disbelief, suspended judgment—is like our chunking of height into a three-fold scheme—tall, short, middling in height. To be tall is to be sufficiently large in one's specific height; but what counts as sufficient is both vague and contextually variable. On the Threshold View, likewise, to believe is to have sufficient confidence; but what counts as sufficient is both vague and contextually variable.

Second, there are strong linguistic reasons to accept the Threshold View as just sketched. After all, predicates of the form 'believes that P' look to be *gradable*. We can append modifiers to belief predicates without difficulty—John fully believes that P. We can attach comparatives to belief predicates without difficulty—John believes that P more than Jane does. And we can conjoin the negation of suchlike without conflict—John believes that P but not fully. These linguistic facts indicate that predicates of the form 'believes that P' are gradable; and that, in turn, is best explained by the Threshold View of coarse belief.[2]

We have, then, three easy pieces:

- The Coarse View
- The Fine View
- The Threshold View.

It is well known they lead to trouble. Henry Kyburg kicked off the bother over four decades ago, focusing on situations in which one can be sure something improbable happens.[3] David Makinson then turned up the heat by focusing on human fallibility.[4] The first issue has come to be known as the Lottery Paradox. The second issue has come to be known as the Preface Paradox. Consider them in turn.

Suppose you know a given lottery will be fair, have one hundred tickets, and exactly one winner. Let L_1 be the claim that ticket 1 loses, L_2 be the claim that ticket 2 loses; and so forth. Let W be the claim that some ticket wins. Your credence in each L-claim is 99%; and your credence in W is thereabouts too. That is just how you *should* spread your confidence. Hence the Threshold View looks to entail that you have rational coarse belief in these claims. After all, you are rationally all but certain of each of them—and the example could be changed, of course, to make you arbitrarily close to certain of each of them. But consider the conjunction

$$\&_L = (L_1 \,\&\, L_2 \,\&\, \ldots \&\, L_{100}).$$

You rationally believe each conjunct. By repeated application of the conjunction rule you should also believe the conjunction. Yet think of the disjunction

$$V_{\neg L} = (\neg L_1 \vee \neg L_2 \vee \ldots \vee \neg L_{100}).$$

You rationally believe *a* ticket will win. That entails the disjunction, so by the entailment rule you should believe it too. Yet the conjunction entails the disjunction is false, so you should believe the disjunction's negation. Hence the conjunction rule ensures you should believe an explicit contradiction: $(V_{\neg L}\ \&\ \neg V_{\neg L})$. That looks obviously wrong.

The reason it does can be drawn from the Threshold and Fine Views. After all, the negation of $(V_{\neg L}\ \&\ \neg V_{\neg L})$ is a tautology. The tautology rule ensures you should lend it full credence. Yet that negation and the contradiction itself are a partition, so the partition rule ensures you should lend the contradiction *no* credence. The Threshold View then precludes rational coarse belief. Our three easy pieces have led to disaster. They entail you both should, and should not, believe a certain claim. For our purposes that is the Lottery Paradox.

Or suppose you have written a history book. Years of study have led you to various non-trivial claims about the past. Your book lists them in bullet-point style: *One Hundred Historical Facts*, it is called. You are aware of human fallibility, of course, and hence you are sure that you have made a mistake somewhere in the book; so you add a preface saying exactly one thing: "something to follow is false." This makes for trouble. To see why, let the one hundred claims be C_1, C_2,..., C_{100}. You spent years on them and have rational credence in each. So much so, in fact, that it makes the threshold for rational coarse belief in each case. You so believe each C-claim as well as your preface. But consider the conjunction of historical claims:

$$\&_C = (C_1\ \&\ C_2\ \&\ \ldots \&\ C_{100});$$

and think of your preface claim P.

Things go just as before: the conjunction rule ensures you should believe $\&_C$. That claim entails ¬P, so the entailment rule ensures you should believe ¬P. The conjunction rule then foists (P&¬P) on you. Its negation is a tautology, so the tautology rule ensures that you should lend the negation full credence. Yet it and the contradiction form a partition, so the partition rule ensures that you should lend the contradiction no credence. The threshold rule then ensures that you should not coarsely believe (P&¬P). Once again we are led to disaster: our three easy pieces entail you both should, and should not, believe a certain claim. For our purposes that is the Preface Paradox.

3. The Main Reactions

Something in our picture must be wrong. Lottery and preface facts refute the conjunction of Coarse, Fine and Threshold Views. Each view looks correct on its own—at least initially—so the Puzzle is to reckon why they cannot all be true.

Most epistemologists react in one of three ways: some take the Puzzle to show that coarse belief and its epistemology are specious; others take it to show that fine belief and its epistemology are specious; and still others take it to show that coarse and fine belief—along with their respective epistemologies—are simply disconnected, that they are unLockean as it were. For obvious reasons I call these the Probabilist, Coarse and Divide-&-Conquer reactions to our Puzzle. They are the main reactions in the literature. Consider them in turn:

(i) The Probabilist reaction accepts the Fine View but denies that coarse belief grows from credal opinion. In turn that denial is itself grounded in a full rejection of coarse belief. The Probabilist reaction to our Puzzle throws out coarse epistemology altogether and rejects any need for a link from it to its *bona fide* fine cousin. How might such a view be defended? Richard Jeffrey puts it this way:

> By 'belief' I mean the thing that goes along with valuation in decision-making: degree-of-belief, or subjective probability, or personal probability, or grade of credence. I do not care what you call it because I can tell you what it is, and how to measure it, within limits...Nor am I disturbed by the fact that our ordinary notion of belief is only vestigially present in the notion of degree of belief. I am inclined to think Ramsey sucked the marrow out of the ordinary notion, and used it to nourish a more adequate view.[5]

The line here simply rejects coarse belief and its epistemology, replacing them with a fine-grained model run on point-valued subjective probability. The resulting position has no room for either the Coarse or Threshold Views.[6]

(ii) The Coarse reaction to our Puzzle accepts the Coarse View but denies that coarse belief grows from credal opinion. In turn that denial is itself grounded in a full rejection of fine belief. The Coarse reaction to our Puzzle throws out fine epistemology altogether and rejects any need for a link from it to its *bona fide* coarse cousin. How might such a view be defended? Gilbert Harman puts it this way:

> One either believes something explicitly or one does not...This is not to deny that in some way belief is a matter of degree. How should we account for the varying strengths of explicit beliefs? I am inclined to suppose that these varying strengths are implicit in a system of beliefs one accepts in a yes/no fashion. My guess is that they are to be explained as a kind of epiphenomenon resulting from the operation of rules of [belief] revision.[7]

The line here simply rejects fine belief and its epistemology, replacing them with a coarse model run on binary belief (i.e., on-off belief). The resulting position says it's a serious mistake to think that sensible confidence makes for rational coarse belief. One does not so believe by investing confidence; and one does not rationally do so by investing sensible confidence.[8]

(iii) The Divide-&-Conquer reaction to our Puzzle accepts Coarse and Fine Views but rejects the Threshold View. The reaction emphasises that coarse and fine belief are central to the production and rationalisation of action. It just sees two kinds of act worth explaining: acts of truth-seeking assertion in the context of inquiry, and practical acts of everyday life. The reaction says that coarse belief joins with desire to explain the former, while fine belief joins with desire to explain the latter. Coarse and Fine Views are both right, on this approach; but the idea that one kind of belief grows from the other is hopelessly wrong. How might this last claim be defended? Patrick Maher puts it this way:

> What is the relation between belief and credence?. . .[I have shown that] no credence short of 100% is sufficient for belief, while a credence of 100% is not necessary for belief. Together, these results show that belief cannot be identified with any level of credence.[9]

The Dive-&-Conquer Reaction holds onto Coarse and Fine Views; but it drops as hopelessly flawed the idea that coarse belief is built from fine belief by a confidence threshold.[10]

4. Critical Discussion

The Fine Reaction says that Threshold and Coarse Views are hopelessly wrong. The Coarse Reaction says that Threshold and Fine Views are hopelessly wrong. Neither is at all plausible, by my lights; for our belief-based practice—in both its coarse and fine guise—simply works too well for either take on belief to be plausible. That practice is exceedingly successful in both guises; and that makes it all but impossible to endorse the idea that either bit of practice is *hopelessly* wrong. The relevant point here is well known in the philosophy of mind. But its thrust for epistemology seems not to be received. One goal of this paper is to help along that process.

So consider: it is doubtless true that our practice of predicting and explaining one another by appeal to coarse belief goes wrong in detail; it is also doubtless true that our practice of predicting and explaining one another by appeal to fine belief goes wrong in detail; but it is very hard to accept that either practice is so hopelessly wrong that *there are no* states of the basic sort mentioned in practice. But that is what the hopeless falsity of Coarse or Fine View entails. If the Coarse View is hopelessly wrong, there are no coarse beliefs even roughly as we suppose, no one does anything because they coarsely believe it will get them what they want, and, as a result, our practice of predicting and explaining one another by appeal to coarse belief is hopelessly flawed to the core. Similarly: if the Fine View is hopelessly wrong, there are no fine beliefs even roughly as we suppose, no one does anything because they are confident it will get them what they want, and, as a result, our practice of predicting and explaining one another by appeal to confidence is hopelessly flawed to the core. If either

View is hopelessly false, *eliminativism* about its favoured kind of belief is true.

This is unacceptable. As Jerry Fodor remarked long ago: if such eliminativism is true, then "practically everything [we] believe about anything is false and it's the end of the world".[11] The hyperbole marks the fact that our belief-based practice—in both its coarse and fine guise—is extremely effective. Just consider an everyday example:[12] a box on your desk rings, you grab it and make noise with your mouth; on that basis I can predict where you will be in one hundred days—at the airport, say—and make sense of you being there—to pick up a friend. How do I manage the predictive feat? I can use coarse or fine belief in the usual way. Sometimes one will seem best for my purpose, other times the other will seem best, depending on context. They both can be used in the normal case. They both are used throughout everyday life.

There is no question but that coarse and fine belief earn their keep in our everyday practice of predicting and explaining one another. That is why eliminativism about either is so hard to take, why we are rationally compelled to endorse the ontic assumptions of Coarse and Fine Views. Those Views may go wrong in detail—and that would mean they go wrong in their norms, of course—but the success of our practice makes it all but impossible to accept that either is hopelessly mistaken, that either goes wrong in its ontology. This means that epistemic perspectives which throw out all but fine belief—like orthodox Probabilism—and epistemic perspectives which throw out all but coarse belief—like most literature on so-called belief revision—fail to do what any view must. They fail to find enough right in practice.

The Fine and Coarse reactions to our Puzzle are therefore unacceptable. We need an epistemology of coarse belief, as well as an epistemology of fine belief; for we speak truly of each kind of belief throughout our causal/predictive life. We're simply obliged, for this reason, to develop an epistemology of each kind of belief as well as a reasoned take on the relation, if any, between them. All too often epistemologists proceed as if this is not so; but that neglects theoretical burdens foisted upon us by practice.

This leaves the Divide-&-Conquer reaction to our Puzzle. The approach finds more right in practice than its more radical cousins, and that is definitely a good thing. But it is still too revisionary, by my lights; for not only do appeals to coarse and fine beliefs work very well in everyday life, they *march in step* when at work. Whenever someone goes to the fridge, say, because they believe that it contains beer, there is a clear and everyday sense in which they go to the fridge because they are confident that it contains beer. And whenever someone goes to the window, say, because they are confident that someone has called out, there is a clear and everyday sense in which they go to the window because they believe that someone has called out. Coarse and fine belief yield everyday action in harmony, marching in step throughout everyday practice.

This cries out for explanation; and it does so in spades on the Divide-&-Conquer approach. After all, that approach has it that confidence and binary belief are quite different things. But then it's surprising that each marches in step with the other as a source of everyday action. Why on earth should *that* be? Why should strong confidence go with binary belief in the production of ordinary acts; and *vice versa*? The Divide-&-Conquer strategy has no internal resource to answer this question. That is one reason to worry about the approach.

Another is more direct still: the strategy does not fully solve the Puzzle with which we began, for it struggles with the Preface. When authors speak to the veracity of their work, after all, the strategy implies

(a) that they should not say it contains mistakes;

and

(b) that they should say it contains no mistakes.

This is because prefaces are truth seeking contexts of inquiry. The Divide-&-Conquer strategy has it that rational claims made in them are closed under conjunction. The preface claim yields conflict with the main text of its book. Such conflict is just what the strategy was meant to avoid, so it must rule them out as rational; and it must go on to insist—when authors are moved to speak on the topic—that they claim to make no mistakes in their work, for that too is entailed by things they believe. Both these pronouncements seem wrong.[13] That yields a strong motivation to look for a different solution to our Puzzle. We need one which does two things at once: finds sufficient truth in Coarse, Fine and Threshold Views; and dissolves both the Lottery and the Preface. The Divide-&-Conquer strategy does neither of these things.

5. Locke's Picture

The Lockean view is less radical than the more popular approaches to our Puzzle canvassed so far. It says the Coarse View is close to right, the Fine View is just fine, and the idea that sensible confidence makes for rational coarse belief is too. The Lockean tinkers with the Coarse picture and accepts the rest of our starting position; and it does so by rejecting a closure condition imposed on coarse belief. Specifically, Lockeans reject the conjunction rule, claiming that common-sense goes slightly wrong with that rule. If one rationally believes P, and rationally believes Q, it is no defect by Lockean lights to withhold belief in (P&Q). Why should we accept such a picture?[14]

Well, for one thing the Threshold View yields an obvious and pleasing story about the causal harmony that exists between coarse and fine belief in everyday practice. It prompts the natural thought, after all, that coarse and fine belief generate action in parallel because they are metaphysically determinable and determinate respectively, because the latter metaphysically makes for the former (as they say). Put another way: the Threshold View prompts the natural idea that coarse and fine belief march in step as the causal source of action because coarse belief is nothing but sufficient confidence. If that were so, coarse and fine belief would causally march in step just as they seem to in practice—they would generate action in parallel; for that is how causal powers of determinable and determinate relate to one another. This strongly suggests that the Threshold View is on the right track.

For another thing, the Threshold View yields an obvious and pleasing story about the rational harmony that exists between coarse and fine belief in everyday practice. It prompts the natural thought, after all, that coarse and fine belief rationalize action in parallel because they are conceptually proximal determinable and determinate respectively. Put another way: the Threshold View prompts the natural idea that coarse and fine belief march in step as rationalizers of action because they are conceptually similar determinable and determinate. This makes for harmony between their rationalzing powers.

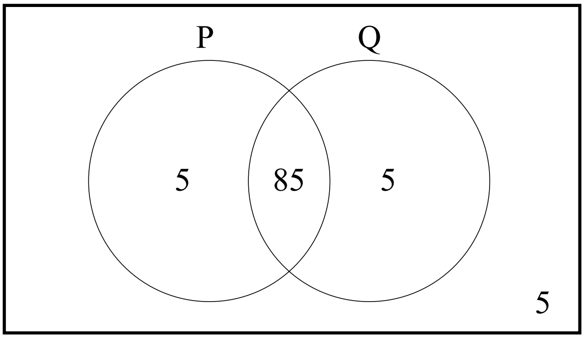
To see why, suppose I have reason to raise my hand: perhaps I want to get the waiter's attention. Wanting in that way is one way of being psychologically; and any way of being psychologically is a way of being as such, a way of existing full stop. Wanting to get the waiter's attention, therefore, is a way of existing full stop. The former metaphysically makes for the latter. But that doesn't mean I have reason to raise my hand just because I exist as such, even though one of my reasons for doing so—namely, my desire to get the waiter's attention—metaphysically makes for existence as such. After all, existence is too far removed, conceptually speaking, from wanting to get the waiter's attention for the former to rationalize action when the latter does so. On the Lockean story, however, coarse and fine belief are not conceptually distal in this way. They are conceptually proximal determinable and determinate, in fact, differing only at the level of grain. This is why the Threshold View ensures that coarse and fine belief rationalize action in step with one another; for that is how conceptually proximal determinable and determinate relate. This strongly suggests that the Threshold View is on the right track.[15]

Only the Lockean approach seems capable of dissolving our Puzzle while finding enough truth in our starting position. Recall it accepts that credence should sum to unity across partitions, that tautologies deserve full credence, that rational coarse belief grows from sensible credence, and that rational coarse belief is closed under entailment. What the view rejects is the conjunction rule.

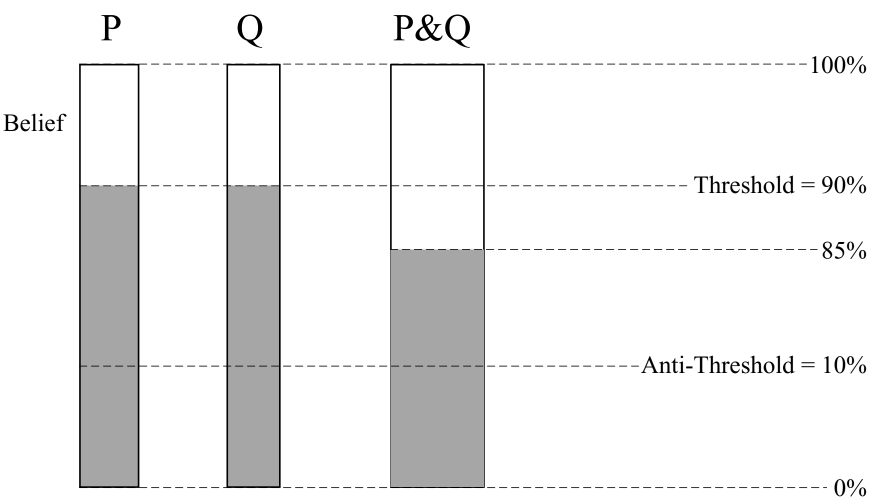
It is easy to see why. Suppose you know this much about a new lottery:

- a single "P" is printed on five tickets
- a single "Q" is printed on five tickets
- the formula "P&Q" is printed on eighty-five tickets
- the remaining five tickets are blank.

Now think of the winner: what is the chance "P" will be on it either alone or in the formula? and what is the chance "Q" will be on it either alone or in the formula? For short: what are the chances of P and Q? Well, the chances look this way:



You should be 90% sure of P, 90% sure of Q, but only 85% sure of their conjunction.[16] Suppose the threshold for belief is 90%. Then you should have belief-level credence in both P and Q but not their conjunction. Your rational credence will flout the conjunction rule. It will look thus:



Rational credence is not preserved by conjunction. For this reason, the Fine and Threshold Views jointly conflict with the conjunction rule. We should reject that rule, reacting to the Lottery and Preface by dropping the rule from which they grow.

Having said that, suppose reasonable credence acts like standard probability.[17] Then the Fine and Threshold Views entail that something very like the conjunction rule holds true; and for that reason, the perspective defended here can find truth in the Coarse View. To see this, let the risk of a proposition be the probability that it is false; and recall that probability of falsity equals one minus probability of truth. It is then easy to prove a lower bound on the risk of a conjunction:

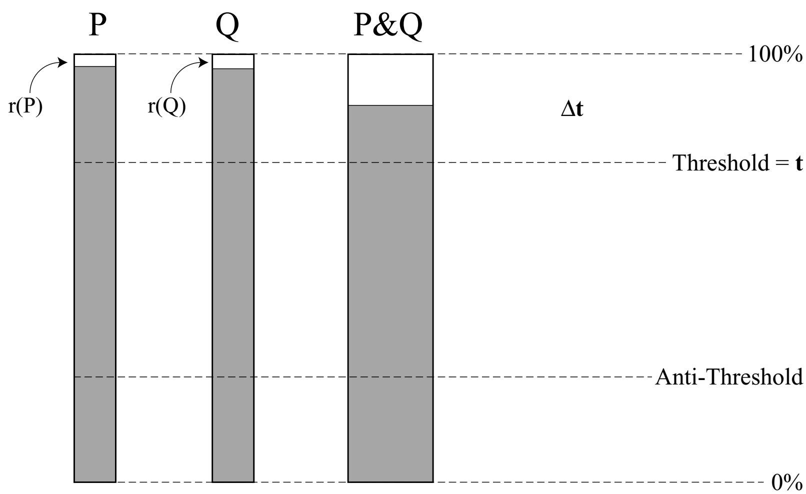
$$r(P\&Q) \leq [r(P) + r(Q)].$$ [18]

The risk of a conjunction cannot exceed the cumulative risk of its conjuncts. Put another way: the chance of going wrong with a conjunction cannot exceed the cumulative chance of doing so with the conjuncts.

Suppose, then, the threshold for coarse belief is **t**. Let Δ**t** be the difference between it and certainty. Suppose the risk of P plus that of Q does not exceed Δ**t**. Then the risk of (P&Q) cannot exceed Δ**t**; and so the probability of (P&Q) must reach the threshold. When probability starts out this way: the Threshold View entails coarse belief in P, and coarse belief in Q, brings with it such belief in (P&Q). If one begins with rational credence, then, instances of the conjunction rule will hold. The approximation requires just this:

$$[r(P) + r(Q)] \leq \Delta t.$$ [19]

In these circumstances, the Fine and Threshold Views entail instances of the conjunction rule. That is why Lockeans can see truth in the rule. They can say instances hold when conjuncts are sufficiently closer to certainty than is the threshold for coarse belief. Instances hold when things look like this:



The Fine and Threshold Views jointly explain why the Coarse View is close to right, why it is not hopelessly false. That strongly indicates the Lockean perspective is on the right track.

But the Lockean perspective does face a pair of serious worries. One is the main concern of those who defend a conjunction rule for coarse belief. The other is the main concern of those who think Probabilism is the only serious game in town. Our next task is to respond to the worries in that order. Each will be given a section.

6. A Worry for Threshold-Based Epistemology: Rational Conflict

The main worry of those who defend a conjunction rule for coarse belief springs from a simple fact: the threshold model permits rational beliefs to conflict. To see this, just think back to the lottery: you believe of each ticket that it will lose, and also that a ticket will win; you know how many tickets there are, so you are in conflict. Without the conjunction rule that conflict stays implicit—you cannot be drawn into believing an explicit contradiction—but the conflict is there in your coarse beliefs all the same. And nothing in the threshold model obliges a shift in view. That model allows you rationally to believe in contradictory things, indeed knowingly to do so.

Many find this unacceptable; and they point to our use of *reductio* in defence of their case. This style of argument is used to damn another's view by exposing tacit conflict in it. This is said to indicate that tacit conflict is ruled out by our practice, that our use of *reductio* shows as much. As Mark Kaplan puts it:

> In putting forth a *reductio* argument, a critic derives a contradiction from the conjunction of a set of hypothesis which an investigator purports to believe. The idea is supposed to be that the critic thereby demonstrates a defect in the investigator's set of beliefs–a defect so serious that it cannot be repaired except by the investigator's abandonment of at least one of the beliefs on which the *reductio* relies. . . But [without the conjunction rule] it is hard to see how *reductios* can possibly swing this sort of weight. [For then] the mere fact that the investigator's set of beliefs has been shown to be inconsistent would seem to provide no reason for her to experience the least discomfort. "The fact that I believe each of the hypotheses in this set," she should respond to her critic, "does not commit me to believing their consequences. So your having shown that a contradiction lurks among those consequences casts no aspersion on my believing the hypothesis in the set."[20]

Against this line of thought, it seems to me, we can make good sense of *reductio* without the conjunction rule. In fact we can make *best* sense of *reductio* without that rule. The *reductio*-based worry has things back-to-front. Or so I will argue.

The key point here is simple: some *reductios* are more potent than others when it comes to dialectically-driven belief revision. The most glaring thing about *reductio*, in fact, is that its punch in this regard is inversely proportional to the number of claims in use. When a *reductio* is drawn from a small number of beliefs, it automatically obliges a shift in view on the part of its victim, irrespective of subject matter or epistemic history. Put another way: when a *reductio* is drawn from a small number of beliefs, it is sufficient on its own to command epistemic movement on the part of its victim, sufficient to mandate a shift in view on their part. When a *reductio* is drawn from a large number of beliefs none of this is true: large *reductios* do not automatically oblige a shift in view irrespective of subject matter or epistemic history. They do not automatically mandate epistemic movement. We feel straightaway compelled to change our view when faced with a *reductio* drawn from a pair of our beliefs, say, but we do not feel that way when faced with one drawn from a hundred of our Lottery beliefs. This dialectical difference is central to our *reductio*-based practice and its phenomenology. The Lockean picture explains it perfectly.[21]

To see this, suppose **B** is a *reductio* set of beliefs $\{B_1,\ldots,B_n\}$. As such **B** is inconsistent. In turn this means certain arguments drawn from it are valid. The negation of B_n follows from the rest of **B**:

$$\begin{array}{c} B_1 \\ \bullet \\ \bullet \\ \bullet \\ \underline{B_{(n-1)}} \\ \therefore \neg B_n \end{array}$$

The negation of B_1 follows from the rest of **B**:

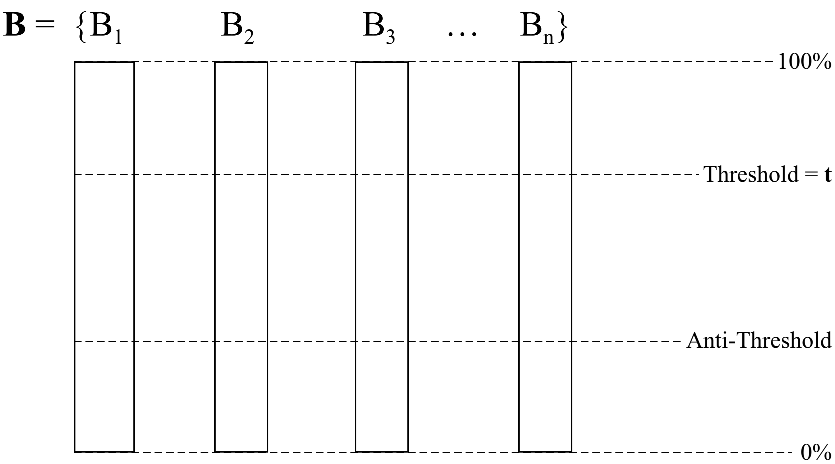
$$\begin{array}{c} B_2 \\ \bullet \\ \bullet \\ \bullet \\ \underline{B_n} \\ \therefore \neg B_1. \end{array}$$

And so on. In general: **B** is a *reductio* set only if the negation of each member is entailed by the others.

There is an easy-to-prove link, though, between validity and risk inheritance. Specifically: the risk of a conclusion cannot exceed the cumulative risk of premises from which it follows.[22] That means **B** is a *reductio* set only if the risk of a member's negation cannot exceed the cumulative risk of other members. Yet risk of negation is probability of truth. Hence we reach the key to *reductio* and its dialectical phenomenology:

(🗝) **B** is a *reductio* set only if the probability of a **B**-member cannot exceed the cumulative risk of other members.

This principle constrains how credence can distribute across *reductio* sets. The Threshold View then explains the phenomenology of *reductio*. Consider the principle's thrust in pictures:



Suppose a *reductio* can be drawn from **B** while credence in each **B**-member meets the threshold for belief. In the event, credence in each **B**-member cannot exceed the cumulative risk of the others. Yet the belief-making threshold is fairly high; so there must a great many beliefs for (🗝) to be true. Put back-to-front: the smaller **B** turns out to be, the less capable credence will be of satisfying (🗝) while reaching the threshold for each **B**-member.

To see how this works in more detail, let's idealise a bit. Assume each **B**-member gets equal credence (as would roughly hold, for instance, were the threshold for coarse belief to be very high). Then (🗝) places an upper bound on each member's credence that is a simple function of the number of beliefs in play:

#(**B**)	Maximal equi-credal **B**-members	
1	0	0%
2	1/2	50.0%
3	2/3	66.6%
4	3/4	75.0%
5	4/5	80.0%
6	5/6	83.3%
7	6/7	85.7%
8	7/8	87.5%

9	8/9	88.8%
10	9/10	90.0%
n	(n-1)/n	

This shows how *reductios* work under present assumptions. When

- the threshold for coarse belief is high, say 90%, and
- one lends equal credence to coarse beliefs,

then,

- a *reductio* cannot be drawn from less than 10 rational coarse beliefs.

If a critic succeeds in producing one from nine or fewer coarse beliefs, therefore, credence is incoherent. Since we are assuming that rational credence is coherent, it follows that the *reductio* demonstrates credal irrationality. The Threshold View then implies that the relevant coarse beliefs are also irrational; and the fewer of them the critic starts with the more potent is her *reductio*.

This is why we feel compelled to change our view when faced with a *reductio* drawn from one or two coarse beliefs, say, but do not automatically feel that way when faced with the Lottery. Since the threshold for coarse belief is relatively high, a *reductio* drawn from a small number of coarse beliefs shows credal incoherence. In turn that shows credal irrationality and thereby coarse belief irrationality. Large *reductios* are not guaranteed to do this. Thus it is that small *reductios* are knock-down on their own—showing fine and coarse belief to have gone drastically wrong—while large *reductios* are no such thing.

When someone is hit with a *reductio*, of course, they have not normally lent equal credence to beliefs used against them; and the lower the threshold turns out to be, the more scope there will be for divergence from that idealising assumption. The details of all this are fairly complex;[23] but the relevant lesson is not: *reductios* are best explained by the Coarse, Fine and Threshold Views together.

7. A Worry for Threshold-Based Epistemology: Pointlessness

Suppose rational degrees of belief act like standard probability functions. In the event, an orthodox version of fine-grained epistemology will be true—namely, Probabilism—but something else will also be true, something perhaps unexpected—namely, a coarse-grained epistemology on which threshold-based belief and its norms derive from credence. After all, one way to have threshold-based coarse belief is to lend credence which exceeds the belief-making threshold; and one way to have rational threshold-based coarse belief is rationally to do so. On the Lockean view: once credence shows up in our epistemology, threshold-based belief shows up as well; for the latter comes for free once the former is in place.

This leads to our last worry about threshold-based epistemology. In my experience it is a worry seen as decisive by most Probabilists; and they continue to see it that way even after admitting that ordinary practice is committed to threshold-based belief and its epistemology. The worry is expressed well by Stalnaker:

> One could easily enough define a concept of belief which identified it with high subjective or epistemic probability (probability greater than some specified number between one-half and one), but it is not clear what the point of doing so would be. Once a subjective or epistemic probability value is assigned to a proposition, there is nothing more to be said about its epistemic status. Probabilist decision theory gives a complete account of how probability values, including high ones, ought to guide behaviour, in both the context of inquiry and the application of belief outside of this context. So what could be the point of selecting an interval near the top of the probability scale and conferring on the propositions whose probability falls in that interval the honorific title 'believed'?[24]

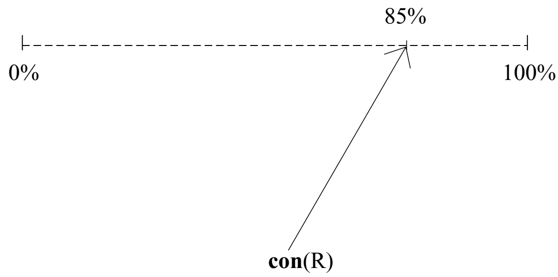
The worry, then, is simple: *if* coarse epistemology springs from its fine cousin via a threshold—if it's Lockean, in our terms—then coarse epistemology is pointless, nothing but residue entailed by real explanatory theory (Probabilism).

This is a serious worry. Lockeans must do more than explain their point of view. They must also explain the point of their view. This will not be trivial, of course, because Stalnaker is right: one *can* easily define a concept of belief by appeal to a threshold; it is *not* clear what the point of doing so would be; and Probabilism *does* seem to give the account—or at least arguably so—of epistemic matters once credence has been assigned. One wonders what's left to be done: what is the *point* of Lockean epistemology?

I close by sketching and defending a direct answer to this question. The answer itself falls out the moral I draw from a series of thought experiments. In turn those thought experiments are not much like ordinary life in their physical set up; but they are very like ordinary life in how evidence impacts on confidence. Once we get clear about that, the point of Lockean epistemology will come into focus; and that point will be seen as non-Probabilist.

Case #1. You are faced with a black box while rationally certain of this much: the box is filled with a huge number of balls; they have been thoroughly mixed; exactly eight-five percent of the balls are red; touching any of them will not affect its colour. You reach into the box, grab a ball, and wonder about its colour. You have no view about anything else relevant to what puzzles you. How confident should you be, in these circumstances, that you hold a red ball?

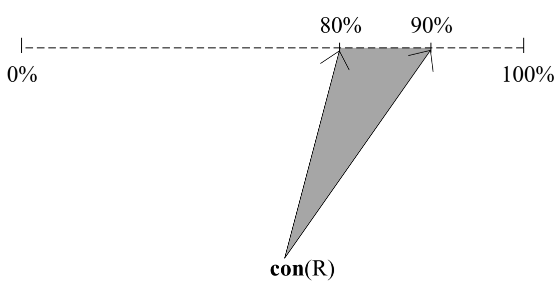
You should be exactly 85% confident, of course. Your confidence in this claim can be modelled, ideally at least, with a point in the psychological space of Probabilism: namely, a credence of .85. On the basis of your evidence you should be exactly 85% sure that you hold a red ball. In pictures:



Case #2. You are faced with a black box while rationally certain of this much: the box is filled with a huge number of balls; they have been thoroughly mixed; exactly 80-to-90% of them are red; touching a ball will not affect its colour. You reach into the box, grab a ball, and wonder about its colour. You have no view about anything else relevant to your question. How confident should you be, in these circumstances, that you hold a red ball?

You should be exactly 80-to-90% confident, of course. Your confidence in this claim cannot be modelled, ideally at least, with a point in the psychological space of Probabilism. Your evidence is too rough for that. Rational confidence demands *more* than a point in credal space. It demands something more like a region instead.

I shall put this by saying that your evidence warrants "thick confidence" in the claim that you hold a red ball; and I shall say that in having such confidence you "occupy" a region of credal space. Just as rational confidence in Case #1 can be modelled with the real number .85, after all, rational confidence in Case #2 can be modelled—as we'll see in a moment—with the interval [.8, .9]. That modelling is motivated precisely because evidence in Case #2 warrants a thick attitude of exactly 80-to-90% confidence in the claim that you hold a red ball. This is why you should occupy a region of credal space, why you should adopt a thick confidence in the claim that interests you, why your take on that claim should be pictured this way:



Now, talk of thick confidence—like that of occupying regions of credal space—is metaphorical, to be sure. But sometimes a metaphor is a good tool to use; and this is one of those times. There are at least five reasons for this. In reverse order of importance they are:

(i) Talk of thick confidence connects humorously and pneumonically with the important fact that evidence in Case #2 is meagre, that it rationally makes for an attitude of relative stupidity. When all you know is that 80-to-90% of balls in the box are red, after all—and you care whether a ball you have grabbed is red—then you are, in a parody British sense at least, "thick" about relevant details. Your evidence warrants only thick confidence in the claim that you hold a red ball.

(ii) Talk of occupying regions of credal space captures the palpable "spread out feel" of the attitude warranted by evidence in Case #2. In some clear sense that attitude is fatter than point-valued subjective probability; and intuitively, at least, that is so because evidence involved in Case #2 is too rough for standard credence, too meagre for such subjective probability. Talk of occupying credal regions is apt because it captures the intuitive feel of the attitude warranted by evidence of this kind.

(iii) Talk of occupying credal regions links directly to the formal model of thick confidence best known in philosophy: van Fraassen's theory of representors. That model gives Probabilism 'a human face'—in Jeffrey's memorable phrase—by modelling thick confidence with sets of probability functions rather than a single probability function. When a rational agent responds to her evidence by lending Φ exactly 80-to-90% confidence, say—when she occupies credal region [.8, .9], in our metaphor—van Fraassen models her take on Φ with a set of probability functions containing, for every number in [.8, .9], a probability function assigning that number to Φ. This set is the agent's representor, and it literally models her thick confidence with regions of credal space; so our talk of occupying such regions connects directly with the representor approach to thick confidence.

(iv) van Fraassen's theory is an obvious extension of Probabilism. Sets of probability functions are used to model an agent's psychological state rather than single probability functions; rational dynamics are developed by applying the update rule of conditionalisation to representors.[25] The resulting view generalises Probabilism in an obvious and pleasing way; and it does so precisely to model thick confidence and its rational dynamics. Unfortunately, the theory yields highly counter-intuitive results about that dynamics. In turn those results flow from an all-too-common technical fact about representors called their 'dilation'.[26] The details of this do not matter for our purposes; but it does matter that the representor approach—as it stands anyway—does not work very well. At present the philosophical literature simply contains no well-functioning *non*-metaphorical model of thick confidence.

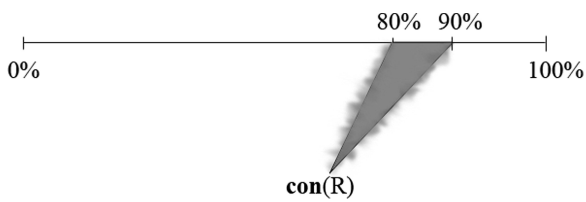
(v) Nothing in the philosophy to follow turns on how thick confidence is formally modelled. All that will matter is that thick confidence is both real and metaphysically as depicted by our metaphors: namely, non-Probabilist. Put another way: all that will matter is that thick confidence is something over and above point-valued subjective probability (i.e., credence).

These five points make it clear that metaphorical talk of thick confidence—like that of occupying regions of credal space—is both well motivated and apt for our purposes. We should be mindful, of course, that such talk *is* metaphorical; but we should not let that prevent us from using it to inspire our work or guide our thought. That shall be my strategy.[27]

Case #3. You are faced with a black box while rationally certain of this much: the box is filled with a huge number of balls; they have been thoroughly mixed; roughly 80-to-90% of them are red; touching a ball will not affect its colour. You reach into the box, grab a ball, and wonder about its colour. You have no view about anything else relevant to your question. How confident should you be, in these circumstances, that you hold a red ball?

You should be roughly 80-to-90% confident, of course. Your confidence in the claim cannot be modelled, ideally at least, with an exact region of credal space; for your evidence is too rough for that tool. Rational confidence seems to demand *more* than an exact region of credal space. It seems to demand something more like a fuzzy region instead.

I shall put this by saying that your thick confidence in Case #3 can be thought of as occupying a vague region of credal space: just as rational confidence in Case #1 can be modelled with the real number .85, and such confidence in Case #2 can be linked to the sharp interval [.8, .9], rational confidence in Case #3 can be linked to the vague interval **v**[.8, .9]. This is the region—or perhaps *a* region—which vaguely begins at .8 and vaguely ends at .9. Your evidence in Case #3 warrants roughly 80-to-90% confidence in the claim that you hold a red ball. This is why you should occupy a vague region of credal space, why you should adopt a fuzzy confidence in the claim that interests you, why your take on that claim should be pictured this way:



Case #4. You are faced with a black box while rationally certain of this much: the box is filled with a huge number of balls; the balls have been thoroughly mixed; touching any of them will not affect its colour; and one more thing. . .(five versions):

(i) A slim majority of balls in the box are red.
(ii) A solid-but-not-total majority of balls in the box are red.
(iii) A very-solid-but-not-total majority of balls in the box are red.
(iv) A very-very-solid-but-not-total majority of balls in the box are red.
(v) Every ball in the box is red.

In each version of the case you reach into the box, grab a ball, and then wonder about its colour. In each version of the case you have no view about anything else relevant to your question. How confident should you be, each time, that you hold a red ball?

Well, it is obvious that you should be more than 50% confident in each version of the case. It is also obvious that your confidence should be weaker in the first version than it is in the second, weaker in the second version than it is in the third, weaker in the third version than it is in the fourth, weaker in the fourth version than it is in the fifth. And it is obvious that your confidence should be maximal in the fifth version: you should be sure that you hold a red ball then. For short, this much is clear:

$$50\% < con_{(i)}(R) < con_{(ii)}(R) < con_{(iii)}(R) < con_{(iv)}(R) < con_{(v)}(R) = 100\%.$$

A bit more specifically: it is clear you should be mildly confident that you hold a red ball in version (i), fairly confident that you do so in version (ii), very confident but not certain that you do so in version (iii), and very, very confident but not certain that you do so in version (iv).

Those fond of sharp confidence will demand an exact level of confidence in each case. They will ask how confident you should be, exactly, in each of them that you hold a red ball. But this is a bad question. It presupposes that Case #4 involves evidence to warrant sharp levels of confidence. That is simply not so: only vague levels of confidence are warranted by evidence in each version of the case. In each of them you should have fuzzy confidence that you hold a red ball, you should lend a fuzzy region of credal space to that claim.

It is of first importance to realize, however, that this is *not* because you are less than ideally rational with your evidence. It is because vague regions of credal space are all that can be got from your evidence. On the basis of your evidence, anyway, perfect thinkers can do no better; for that evidence is vague through and through. Fuzzy confidence is all that can be got from it. That evidence rationally makes for no more than fuzzy confidence.

This springs from a very important normative fact: *evidence and attitude aptly based on it must match in character*. When evidence is essentially sharp, it warrants a sharp or exact attitude; when evidence is essentially fuzzy—as it is most of the time—it warrants at best a fuzzy attitude. In a phrase: evidential precision begets attitudinal precision; and evidential imprecision begets attitudinal imprecision.

Moreover, it cannot be said with authority where warranted fuzzy regions of credal space begin or end in Case #4. That could be done in Case #3, of course; but that was because it involved a sharp credal region fuzzed up with vagueness—a sharp credal region *mit slag*. Case #4 is much more like everyday life, involving vague evidence all the way down, vague evidence through and through. Credal regions warranted by this kind of evidence are fuzzy, like in Case #3; but unlike that case no one can say—with authority at least—where those regions vaguely begin or vaguely end.

Normally quotidian evidence is vague through and through: we must decide what to think on the basis of essentially fuzzy evidence. That is why most of the time we should lend vague regions of credal space to claims of interest, why bread-and-butter rationality is fully fuzzy rather than sharp. But to repeat: this is not because Probabilist agents are hyper-ideal in relation to regular folk; it is because epistemic perfection demands *character match* between evidence and attitude: when the former is fuzzy, the latter should be too; when the former is sharp, the latter should be too.

This demand leads to the *raison d'être* of threshold-based epistemology. To see this, recall Stalnaker's claim that Probabilism gives a full view of epistemic matters once credence has been assigned. Assume he is right about that.[28] In the event, threshold-based epistemology can look to serve no purpose. *If* Probabilism is a complete account of rational credence, threshold-based epistemology seems to be a redundant tag-along at best. Its point must be clarified in a way consistent with the idea that Probabilism is the full story of credence.

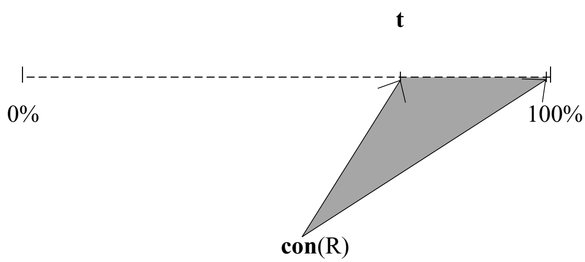
This can now easily be done.

The first point to note is that everyday evidence does not normally make for credence. As we have seen, it does not normally make for sharp levels of confidence at all; and nor does it normally make for hyper-thin ones like point-valued credence. In everyday life, at least, our evidence is normally like Case #4: it warrants fuzzy thick confidence. Credence is neither fuzzy nor thick, differing twice over from attitudes normally warranted by everyday evidence. Those attitudes are fuzzy rather than sharp, thick rather than hyper-thin. In a nutshell: everyday evidence tends to rationalise fuzzy regions of credal space.

The second point to note is that when fuzzy confidence is thick enough, and fuzzy confidence is strong enough, there is simply *no difference* between it and threshold-based coarse belief. After all, when fuzzy confidence is thick enough—say around five to fifteen percent of the scale, depending on context—and fuzzy confidence is strong enough—toward the certainty end of the sale, of course—lending that confidence to a claim functions exactly like believing it in a threshold-based way. Yet functional identity entails type identity of attitude, for attitudes are functionally individuated; so when fuzzy confidence is thick enough, and fuzzy confidence is strong enough, it follows that lending such confidence to a claim is identical to believing it in a threshold-based way. The key thought here is easy to state: strong thick fuzzy confidence is identical to threshold-based belief.

Think of it this way: fix a confidence threshold for coarse belief and then consider everyone who believes Φ relative to it. Some will do so by lending Φ sharp confidence above the threshold—both thick and thin—others will do so by lending Φ fuzzy confidence above the threshold—both thick and thin. Each way of managing the task corresponds to its own functional property; for each way of doing so is its own type of propositional attitude; and attitudes are individuated functionally. As a result: every sharp confidence above the threshold—both thick and thin—corresponds to its own functional property; and every fuzzy confidence above the threshold—both thick and thin—does so as well. In turn these functional properties each make for threshold-based belief in Φ; and since there are countless of them, there are countless ways to manage the task. But one of those ways will be special. It will make for an attitude specifiable in two different idioms: one of them will be drawn from coarse-grained psychology, the other will be drawn from fine-grained psychology.

To see this, consider the fuzzy region of credal space from the threshold to certainty itself. This will be the belief-making region of credal space. Call it the "bel-region" to keep that in mind. Then note that one might lend a claim *exactly* that region of credal space; and if that occurred, one would manifest a functional property shared by all and only threshold-believers in Φ. That functional property grounds thick fuzzy confidence stretching—vaguely, of course—from the threshold to certainty; but it also makes for threshold-based belief. Bel-region confidence is functionally grounded exactly like threshold-based belief. Since attitudes are individuated functionally, it follows that lending bel-region confidence and threshold-based belief are one and the same attitude. Lending bel-shaped thick fuzzy confidence is identical to believing in the threshold-based way. Occupying the bel-region of credal space is identical to threshold-based believing.[29] In pictures: ¨



Here '**t**' marks sharply what is meant to be vague (and contextually variable): the belief-making threshold. From **t** to certainty is the bel-region of credal space. Occupying it occurs when one lends bel-level confidence to a claim.

When that happens, the psychology of coarse and fine epistemology *overlap*. They both contain the attitude called 'belief' in everyday life and 'bel-level confidence' here. Two names stand for one attitude. The first name fits into a three-fold scheme linked to coarse-grained epistemology. The second fits into a countless-fold scheme linked to fine-grained epistemology.[30] The schemes overlap because threshold-based belief is the same attitude as bel-shaped confidence.

This means coarse and fine epistemology overlap in their *norms* as well as their psychology. They share norms for their common attitudes. In fine-grained epistemology, those norms will be said to concern thick and particularly strong/weak confidence (i.e., belief- and disbelief-shaped confidence). In coarse-grained epistemology, those norms will be said to concern belief and disbelief as such. But this will involve the same bit of theory twice over, namely, norms for attitudes normally warranted by everyday evidence. This is why the intersection of coarse and fine epistemology—and thus coarse epistemology itself—is of first theoretical importance. It contains norms for bread-and-butter rationality.

Probabilism may give a full view of rational credence, but it does not give a full view of fine-grained epistemology. If it did, ideal agents would always assign a point-valued subjective probability to questions of interest. It is both clear and widely recognized that this is not so. Often evidence is too coarse for such probability; and when that happens epistemic perfection rules out credence, demanding instead some kind of region of credal space. What we should frequently do—if we're ideally to respect our evidence—is adopt a fine-grained state which is also a coarse-grained state. We should adopt a fine-grained state which functions—both metaphysically and normatively—as a coarse-grained state functions—both metaphysically and normatively. On the basis of everyday evidence, we should often adopt an attitude at the heart of both coarse and fine epistemology. That is why Lockean epistemology is of theoretical moment even if Probabilism is the full story about rational credence. Lockean epistemology captures the heart of everyday rationality.

Notes

[1] A partition is a collection of claims guaranteed by logic to contain exactly one true member. A credence is an exact percentage of certainty (e.g., 50%, 75%, etc.).

[2] Stanley (2005).

[3] Kyburg (1961), p. 197.

[4] David Makinson (1965).

[5] Jeffrey (1970), pp. 171–2.

[6] Colin Howson and Peter Urbach defend a Probabilist line like this in their (1989), ch. 3.

[7] Harman (1986), p. 22.

[8] John Pollock and Joe Cruz defend a line like this in their (1999).

[9] Maher (1993), p.135. I have put talk of credence for that of subjective probability, and talk of belief for that of acceptance (in line with Maher's p.130).

[10] For a perspective like this see chapter 3 of Kaplan (1996), chapter 6 of Maher (1993), or chapter 5 of Stalnaker (1984).

[11] Fodor (1990), p. 156.

[12] Fodor (1987) p. 3ff.

[13] For a good discussion of this see Christensen (2004).

[14] Foley (1992) defends a Lockean view. The position is also endorsed by Field (2003), and Schiffer (2003), ch. 5.

[15] For more on the causal and rational harmony between determinable and determinate see Yablo (1992) and (1993). For similar thoughts—in embryonic form and developed independently—see Sturgeon (1994).

[16] This is because you are rationally certain about the chances just mentioned and you have no other relevant data about P or Q; so you should set your credences in them equal to their chances of being true. The classic discussion of this is Lewis (1986).

[17] That is a stronger assumption than anything made so far, obliging sensible conditional credence to line up with its unconditional cousin in the usual ratio way. We will assume this from now on.

[18] **Proof:** Propositional logic ensures P is equivalent to [(P&Q) v (P&Q̸)], so probability theory ensures cr(P) equals cr[(P&Q) v (P&Q̸)]. The disjuncts are exclusive, so probability theory ensures cr(P) equals [cr(P&Q) + cr(P&Q̸)]. From logic we know (P&Q̸) entails Q̸, so probability theory ensures cr(P&Q̸) ≤ cr(Q̸). But that theory also ensures cr(Q̸) equals 1 minus cr(Q). Algebra then yields a lower bound for rational credence in (P&Q): [cr(P) + cr(Q) – 1] ≤ cr(P&Q). This (plus algebra and the definition of risk) leads to the lower bound in the text. And the result can be generalized by induction on the length of a conjunction.

[19] Ernest Adams has done more than any other to make clear why rules such as the conjunction rule strike us as intuitively correct despite foundering within a probabilistic setting. For a good introduction to that work see his (1998).

[20] Kaplan (1996), pp. 96–97. Similar sentiments are expressed in Maher (1993), ch. 6.

[21] I do not say that we never feel obliged to shift our view when hit with a large *reductio*. As Mark Kaplan (2002) points out, on some occasions we do feel obliged to react in that way when hit with a large *reductio*. On other occasions, though, we do not feel so obliged when hit with such a *reductio*—as the Lottery example makes clear—and that is what distinguishes large *reductios* from small ones. We always feel obliged to shift our view when hit with a small *reductio*. We do not always feel obliged to shift our view when hit with a large *reductio*. Key factors in determining how it will go are the subject matter involved and our epistemic history with it. See chapter 5 of *Epistemic Norms* (forthcoming Oxford University Press) for further discussion.

[22] **Proof:** Suppose C is entailed by P1,. . ., Pn. Then (P1&. . .& Pn) implies C. So cr(P1&. . .& Pn) ≤ cr(C). So r(C) ≤ r(P1&. . .& Pn). We have already proved, though, that the risk of a conjunction cannot exceed the cumulative risk of its conjuncts. So r(C) ≤ [r(P1) +. . .+r(Pn)]. This result forms into the backbone of Probability Logic; and its most fecund use—in philosophy, anyway—comes through its application to the theory of conditional belief and its logic. See Adams (1975), Edgington (1994), and chapter 7 of my *Epistemic Norms*.

[23] A nice discussion of this technical issue can be found in Hawthorne and Boven (1999).

[24] Stalnaker (1984), p.138. I have put talk of 'belief' in for that of 'acceptance'; but nothing is lost in the translation.

[25] Probabilism models an agent's rational degrees of belief at a time with a single probability function. It then says rational shift in view should occur in accordance with the update rule of conditionalisation. That rule says, in turn, that when an agent starts out modelled by an initial probability function P_{old}, which happens to lend some-but-not-full probability to Φ, and then the agent becomes certain of Φ, her new probability function P_{new} should equal her old one conditional on Φ. In other words, for any claim Ψ: $P_{new}(\Psi)$ should equal $P_{old}(\Psi/\Phi)$ in these circumstance. Conditionalisation is then applied to representors by applying it to their members for which it is defined.

[26] Intuitively, the dilation of a representor occurs when a thick confidence in Φ at one moment—which does *not* stretch from no confidence to full confidence—turns into a thick confidence in Φ at the next moment—which *does* stretch out in that way—simply because the agent learns something intuitively irrelevant to Φ. Edifying technical discussion of dilation can be found in Siedenfeld and Wasserman (1993). See also Heron, Seidenfeld and Wasserman (1997). Philosophical discussion of dilation can be found in van Fraassen's (1990) and (2005).

[27] The need for thick confidence has long been recognized. Classic recent philosophical discussion of the notion can be found in Hacking (1975), Levi (1974) and Jeffrey (1983). More recent philosophical discussion of thick confidence can be found in Joyce (2005), Kaplan (1996),

and Maher (1993). van Fraassen first presented his representors in his (1985); see also his (1987). It is interesting to note, moreover, not only that the first major work on the mathematics of thick confidence—due to Keynes—came *prior* both to Ramsey's creation of Probabilism and to Kolmogorov's classic treatment of its point-valued mathematics, but also that Ramsey's Probabilism was developed in *reaction* to Keynes' work. After all, Keynes worked on this topic in the early part of the twentieth century while Ramsey's reaction came in 1926, and Kolmogorov's classic text appeared only five years later. See Keynes (1921), Ramsey (1978), and Kolmogorov (1933). Thick confidence, in our sense, is basically Ramsey's subjectivism stripped of its point-valued mathematics and the overly precise metaphysics meant to be modeled by it. For recent technical discussion of the idea see Walley's encyclopedic (1991) and Halpern (2003). The very latest work on the topic can be found at The Imprecise Probabilities Project online: http://ippserv.rug.ac.be/

[28] In fact I do not think that he *is* right about that; for Probabilism mishandles conditional thought, forcing it into propositional mode. See chapter Four of *Epistemic Norms*.

[29] And the same holds true, *mutatis mutandis*, for threshold-based disbelief and suspended judgment. When fuzzy confidence is thick enough—say around five to fifteen percent of the scale, depending on context—and fuzzy confidence is weak enough—toward the no-confidence end of the scale, of course—lending such confidence is identical to disbelieving in a threshold-based way. And when fuzzy confidence is thick enough—say between seventy and ninety percent of the scale, depending on context—and fuzzy confidence is middling enough—in the middle of the scale, of course—lending such confidence is identical to suspending judgment in a threshold-based way. All this is discussed much more fully in Chapter Six of *Epistemic Norms*. There it is argued, in fact, that the threshold-based identities of belief and disbelief are correct, but the analogue claim about suspended judgment is incorrect. Examining why in detail would take us too far away from present concerns.

[30] The latter scheme can be got by *adding* levels of thick confidence—both sharp and fuzzy–to the credal space of Probabilism. The result is a well-motivated fine-grained psychological space, one on which full-dress epistemology should be run. I am thus *not* recommending the rejection of point-valued subjectivity probability within epistemology, but rather its supplementation.

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