Bayes or Bust?
A Critical Examination of Bayesian Confirmation Theory

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3 Dutch Book and the Axioms of Probability

Rather than simply assuming that degrees of belief are regimented by the principles of probability, one could try to exploit the interpretation of probability as degree of belief as a means of getting a justification for the probability axioms. We saw in chapter 1 that Thomas Bayes took this tack by using the connection between degrees of belief and betting behavior. Ramsey (1931) and de Finetti (1937) followed a related tack with their Dutch-book strategy, although they were apparently unaware of the details of Bayes's work, which contains, as we have seen in chapter 1, intimations of Dutch book. The presentation given here follows Shimony 1955.

By a *bet* on $A \in \mathscr{A}$ let us understand a contractual arrangement between a bettor and a bookie by which the bettor agrees to pay the bookie the amount $\$b$ if A turns out to be false and the bookie agrees to pay the bettor $\$a$ if A turns out to be true. The sum $\$(a + b)$ is called the *stakes* of the bet, and the ratio b/a is called the bettor's *odds*. If Pr is the bettor's degree-of-belief function, the expected monetary value of the bet for him is $\$a \times \Pr(A) - \$b \times \Pr(\neg A)$. The bet is said to be *fair* (respectively, *favorable*, *unfavorable*) to the bettor according as the expected value is zero (respectively, positive, negative). Using the negation principle (P1), the condition for a fair bet comes to $\Pr(A) = b/(a + b)$. This ratio is called the bettor's *fair betting quotient.*

The idea of the Dutch-book argument is to turn this construction around to produce a justification of the probability axioms: assume that degree of belief functions as a fair betting quotient and then show that something very nasty will happen if the degrees of belief fail to conform to the probability axioms. Thus if $\Pr(A) = r$ is your degree of belief in A, then (the story goes) you should be willing to bet on A on the terms in table 2.1. S is allowed to be either positive or negative, which means that you are

Table 2.1
Terms for betting on A

	Pay	Collect	Net
A false	rS	0	$-rS$
A true	rS	S	$(1-r)S$

Note: S stands for the stakes.

required to accept either end of the bet. If you do enter such an arrangement, the nasty thing that threatens is Dutch book, a finite series of bets such that no matter what happens, your net is negative (a violation of what is called *coherence* for degrees of belief). The *Dutch-book theorem* shows that if any one of the axioms (A1) to (A3) is violated, then Dutch book can be made. The *converse Dutch-book theorem* shows that if (A1) through (A3) are satisfied, then Dutch book cannot be made in a finite series of bets. This converse is crucial to the motivation for conforming degrees of belief to the principles of probability, for if such a conformity were not guarantee against Dutch book, the threat of Dutch book would not be a very effective inducement to conformity. Only the proof of the Dutch-book theorem will be sketched here. The interested reader can consult Kemeny 1955 and Lehman 1955 for the converse.

To establish that (A1) is necessary to avoid Dutch book, suppose that $\Pr(A) = r < 0$. Choose $S < 0$ and note that the net is negative whether or not A is true. Similarly, if $\Pr(A) = r > 1$, choosing $S > 0$ leads to a loss, come what may. We can now establish that (A2) is necessary to avoid Dutch book. For suppose that $\Pr(A) = r \neq 1$ even though $\models A$. By the previous results, $0 \leqslant r \leqslant 1$. Choosing $S < 0$ then leads to a loss in case A is true, which is the only possible case. Finally, to show the necessity of (A3), suppose that $\models \neg(A \,\&\, B)$ and consider a series of three bets: one on A with a betting quotient $\Pr(A) = r_1$ at stakes S_1, one on B with a betting quotient $\Pr(B) = r_2$ at stakes S_2, and one on $A \vee B$ with a betting quotient $\Pr(A \vee B) = r_3$ at stakes S_3. There are three possible cases to consider (table 2.2). The theory of linear equations then shows that the stakes can be chosen so that the nets are all negative unless $r_3 = r_1 + r_2$, i.e., unless (A3) holds.

If regarded as a definition, the formula given in section 1 for conditional probability does not stand in need of a justification. But as in de Finetti 1937, the notion of the conditional probability of B on A can be introduced

Table 2.2
Net payoffs for the three bets taken together

	Net
A true, *B* false	$(1 - r_1)S_1 - r_2 S_2 + (1 - r_3)S_3$
A false, *B* true	$-r_1 S_1 + (1 - r_2)S_2 + (1 - r_3)S_3$
A false, *B* false	$-r_1 S_1 - r_2 S_2 - r_3 S_3$

as a primitive and then operationalized in terms of a bet on *B* conditional on *A*, the terms of which specify that if *A* obtains, a standard unconditional bet on *B* is in effect, whereas if *A* fails, the bet is called off. Then (the story goes) $\Pr(B/A)$ should be the agent's critical odds for this conditional bet. The agent is now offered three bets: a standard bet on *A*, a standard bet on *B* & *A*, and a bet on *B* conditional on *A*. It is left as an exercise to show that unless $\Pr(B/A) \times \Pr(A) = \Pr(B \,\&\, A)$, stakes can be chosen for the three bets so that the agent has a sure net loss. This argument does *not* justify the rule of conditionalization, which requires a different argument (see section 5 below).

The Dutch-book justification for continuity is not so pretty, and this is perhaps one of the reasons it plays no role in the Bayesianism of Ramsey, de Finetti, and Savage.[6] To Dutch-book a violation of (C) or (A4), which is not also a violation of (A1) through (A3), requires laying an infinite series of bets. But if I were to risk the same finite amount, no matter how small, on each of these bets, then I would have to have an infinite bankroll, an impossible dream. And if the dream should come true, I would not care one whit about losing a finite or even an infinite sum if, as can always be arranged, I have an infinite amount left over. To remedy this defect, we can imagine that the bettor accepts an infinite series of fair bets but that the total amount he risks is finite; e.g., he risks \$(1/2) on the first bet, \$(1/4) on the second, \$(1/8) on the third, and so on. With this setup, Adams (1961) shows that a sure loss results from a violation of the general continuity axiom (C) (see also Spielman 1977).

4 Difficulties with the Dutch-Book Argument

Qualms about the Dutch-book justification of the probability axioms are so numerous and diverse that it is hard to classify them. For future reference I note that when the requirement of logical omniscience is dropped, as it must be for realistic agents, the situation becomes more complicated; this matter is discussed in chapter 5. For the present context, which takes logical omniscience for granted, I begin with three miscellaneous qualms. First, the Dutch-book construction for countable additivity involves, in Ernest Adams's words, "extremely unrealistic systems" (1961, p. 8). For those who insist that degrees of belief must be operationalized in terms of economic transactions, this constitutes a reason to reject countable additivity. (Thus it is not surprising that countable additivity plays no role in de Finetti's personalism.) But for those of us who reject operationalism and behaviorism and insist that countable additivity is needed, the difficulty is a shortcoming of the Dutch-book construction. Second, the requirement that the agent be willing to take either side of the bet (i.e., the stakes S may be either positive or negative) may not be satisfied by actual gamblers, and in any case it already assumes the negation principle.[7] Third, a Bayesianism that appeals to both Dutch book and strict conditionalization is on a collision course with itself. The use of strict conditionalization leads to situations where $\Pr(A) = 1$ although $\not\models A$. As a result, something almost as bad as Dutch book befalls the conditionalizer; namely, she is committed to betting on the contingent proposition A at maximal odds, which means that in no possible outcome can she have a positive gain and in some possible outcome she has a loss (a violation of what is called *strict coherence*). It is too facile to say in response that this is a good reason for abandoning strict conditionalization in favor of Jeffrey conditionalization or some other rule for belief change; for all the results about merger of opinion and convergence to certainty so highly touted in the Bayesian literature depend on strict conditionalization (see chapter 6).

A more basic worry harkens back to Bayes's insistence that probability as a betting quotient be attached to "events," i.e., decidable propositions (see chapter 1). Bets on the outcome of the Kentucky Derby are one thing, bets on scientific hypotheses are quite another. A hypothesis with the quantifier structure $(\exists x)(\forall y)Rxy$ can be neither verified nor falsified by finite means. Thus a bet on such a hypothesis turns on a contingency that can never be known for certainty to hold or to fail, and so the parties to the bet have no sure way to settle the matter. To try to settle the bet by appeal to the probable truth or falsity of the hypothesis runs afoul of the fact that the parties can and often do disagree on whether the hypothesis is probably true. But if the bet is never paid off, fear of being bilked disappears.

The response to this worry might be that bookies wearing wooden shoes, money pumps, etc. are just window dressing. The underlying assumption is that degrees of belief are manifested in preferences over the kinds of bets described in section 3. This assumption granted, the Dutch-book construction stripped of its decoration shows that the failure of degrees of belief to conform to the probability calculus results in a structural inconsistency in the individual's preferences. Suppose that the individual is nonsatiated in that she prefers more money to less. Then if this person violates (A1) or (A2), the Dutch-book construction reveals that she is literally inconsistent with herself, since she prefers the certainty of handing over some $\$\varepsilon > 0$ to the status quo, despite her professed nonsatiation. In the case of (A3) the argument is more involved, since it appeals to another principle, "the package principle"; to wit, a person's preferences are inconsistent if there is a finite series of bets such that she regards each as preferable to the status quo while at the same time she regards the status quo as preferable to the package of bets. If this hypothetical agent violates (A3), we proceed to construct a finite series of bets each of which she finds favorable. By the package principle, she should then find the package favorable. But the package is shown to be equivalent to handing over $\$\varepsilon > 0$, which contradicts nonsatiation. Note that on this reading the Dutch-book construction does not justify strict coherence, i.e., the requirement that $\Pr(A) = 1$ only if $\models A$, which I take to be a mark in favor of this reading.

Schick (1986) has questioned the normative status of the package principle. Its plausibility, he argues, rests on accepting the notion of value additivity, which holds that the value of the package of bets is the sum of the values of the individual bets. But, Schick claims, an agent who refuses to conform her degrees of belief to the probability axioms may read the Dutch-book construction as a reason to reject value additivity. Schick's objection may not at first seem very moving, but it gains force in the context of the sequential decision making that comes into play in the attempted diachronic Dutch-book justification for conditionalization (see section 6).

Although the above reconstrual of the Dutch-book construction is a step forward, it is still too closely tied to the behavioristic identification of belief with dispositions to place bets. Once it is admitted that betting behavior is only indicative of, and not constitutive of, underlying belief states, it must also be admitted that belief and behavior are mediated by many factors and that these factors can weaken to the breaking point the simpleminded linkage assumed in the Dutch-book construction. In poker, for example, betting high may be a good way to scare off the other players and win the pot (see Borel 1924). And generally, a knowledge of the tendencies of opponents may make it advisable to post odds that differ from one's true probabilities (see Adams and Rosenkrantz 1980).[8]

Two responses can be made to this complaint. First, one can drop the Dutch-book approach in favor of a justification of the probability axioms that focuses directly on the nature of belief and the cognitive aims of inquiry and eschews altogether preferences for goodies, monetary or otherwise. Some candidates for such a justification will be examined in the next section. Second, one can continue to push the Dutch-book approach by taking into account in a more systematic manner the preference structure of the agent. I will follow this theme in the remainder of this section.

The opening melody of this theme is that the Dutch-book construction rests on the assumption that utility is linear with money, or equivalently, that agents are risk neutral, an assumption known to be false for many if not most real-world agents.[9] To illustrate the complications that can arise in trying to use betting behavior to elicit degrees of belief for such real-world agents, let us analyze from the point of view of expected-utility theory the elicitation device Bayes himself used. Let $\$q$ be the maximum amount the agent is willing to pay for a contract that awards $\$r$ if A is true and \$0 otherwise. If U is the agent's utility function and $\Pr(dw/A)$ and $\Pr(dw/\neg A)$ are the agent's conditional probability distributions for wealth exclusive of the contract prize, then a little algebra shows that the expected-utility hypothesis implies that the agent's degree of belief in A is

$$1\Bigg/\left[1+\frac{\int(U(w+r-q)-U(w))\Pr(dw/A)}{\int(U(w)-U(w-q))\Pr(dw/\neg A)}\right]$$

(see Kadane and Winkler 1987). If the agent is risk neutral, i.e., if U is linear, then the degree of belief is seen to be equal to q/r, as Bayes thought. If $\Pr(dw/A)=\Pr(dw/\neg A)$ (i.e., the agent's wealth apart from the contract payoff is not probabilistically dependent on A) but the agent is not risk neutral, then $\Pr(A)$ will differ from q/r: if the agent is risk-averse, q/r will understate $\Pr(A)$, while if she is risk-positive, q/r will overstate $\Pr(A)$. And if $\Pr(dw/A)\neq\Pr(dw/\neg A)$, q/r is an even more distorted measure of $\Pr(A)$.

The moral is that the direct elicitation of degrees of belief by betting behavior is doomed to failure. Degrees of belief and utilities have to be elicited in concert. In the standard developments of this concerted elicitation the aim is to show that preferences satisfying (what are taken to be) rationality constraints can be represented in terms of expected utility, with the probabilities being uniquely determined and the utilities determined up to positive linear transformations. But the alleged rationality constraints are open to challenge (see, for example, the paradoxes in Allais 1953 and Ellsberg 1961). Moreover, when the utilities are dependent not just on the prizes but also on the propositions whose utilities are being elicited, then the probabilities may not be uniquely determined (see Schervish, Seidenfeld, and Kadane 1990 and Seidenfeld, Schervish, and Kadane 1990). Here I must break off the discussion, since I have strayed beyond the scope of this work.

6 Justifications for Conditionalization

Dutch-book justifications can be given for both strict conditionalization (Teller 1973, 1976) and Jeffrey conditionalization (Skyrms 1987).[11] To consider the former, suppose without any real loss of generality that upon learning E the agent shifts from $\Pr_{\text{old}}$ to $\Pr_{\text{new}}$, where $y = \Pr_{\text{old}}(A/E) - \Pr_{\text{new}}(A) > 0$ and $x = \Pr_{\text{old}}(A/E) > 0$. The diachronic Dutch bookie first sells the agent three bets b_1: [\$1; A & E], b_2: [\$$x$; $\neg E$], and b_3: [\$$y$; E], at what the agent computes to be their fair values. (Recall that [\$$z$; C] stands for the contract that pays \$$z$ if C obtains and \$0 otherwise.) If E proves to be false, the agent has a net loss of \$$y\Pr_{\text{old}}(E)$. On the other hand, if E turns out to be true, the bookie buys back from the agent the bet b_4: [\$1; A] for its then expected value to the agent (\$$\Pr_{\text{new}}(A)$ = \$$(\Pr_{\text{old}}(A/E) - y)$). The agent then has a net loss of \$$y\Pr_{\text{old}}(E)$, regardless of whether A obtains.

We can assess this argument for conditionalization in the light of the distinction drawn above in section 4 between two readings of the Dutch-book construction. If the central concern is to escape being systematically bilked by a bookie, there is a simple solution that doesn't commit you to conditionalization: don't publicly announce your strategy for changing belief in the face of new evidence. If you are worried about clairvoyant bookies who can read your mind, then don't make up your mind in advance; just wait to see what evidence comes in and then wing it. (This is, in fact, what many of us do in practice.) This will make you proof against systematic bilking, save by those bookies who have the ability to foresee your future belief states. But from such precognitive bookies not even good Bayesian conditionalizers are safe. Of course, if you do not conditionalize, there will be a hypothetical lucky bookie who by chance rather than system hits on a series of bets that guarantees you a net loss, but then even if you do conditionalize, there will be a hypothetical lucky bookie who takes you for a loss.

On the more pristine reading of the original synchronic Dutch-book construction, the bookies in wooden shoes were only window dressing, and what was really being revealed (so the story went) was a structural inconsistency in the preferences of an agent who did not conform her degrees of belief to the probability calculus. In applying this reading to the diachronic setting, we need to divide cases. Consider first the case of an agent who eschews preset rules for changing degrees of belief. In this instance it is hard to see how the charge of inconsistency can legitimately be leveled. For how can such an agent's preferences over bets at t_1 be inconsistent with her preferences over bets at t_2 any more than her preferences over wines at t_1 can be inconsistent with her preferences over wines at t_2? Perhaps in response it will be urged that without melding together preferences at different times to form an integrated whole, it wouldn't be proper to speak of an enduring agent. That is certainly true, but surely the requirements for personal identity over time cannot be taken to entail rationality constraints—and conditionalization is allegedly such a constraint—since a person who behaves irrationally does not cease to be a person.

The agent who has adopted a rule for belief change is more open to the charge of inconsistency, since she has already committed herself at t_1 to what her preferences over bets will be at t_2. It would then seem that we can apply at t_1 the package principle introduced in the discussion of synchronic Dutch book: if an agent prefers each of a finite series of bets to the status quo, then she also prefers the package of bets to the status quo. To make this principle yield the desired consequence in the present setting, 'prefer' must be taken to mean prefer when the decision is viewed as an isolated one, which is the tacit understanding in effect when the critical odds for a bet on A are used to elicit the agent's degree of belief in A. But an agent who is not a conditionalizer can satisfy the package principle by taking 'prefer' to mean prefer when the decision to accept or reject the bet is placed in the context of a sequential decision problem. If we view the diachronic Dutch-book construction as a sequential decision process, the decision tree looks as in figure 2.1. The principles of rational decision making require that at decision node 1 the agent face up to what she knows about what her preferences will be at node 2, should she get there (see Seidenfeld 1988). She knows that at node 2 the tiniest premium will lead her to prefer to sell back to the bookie the bet on A, and she sees that in

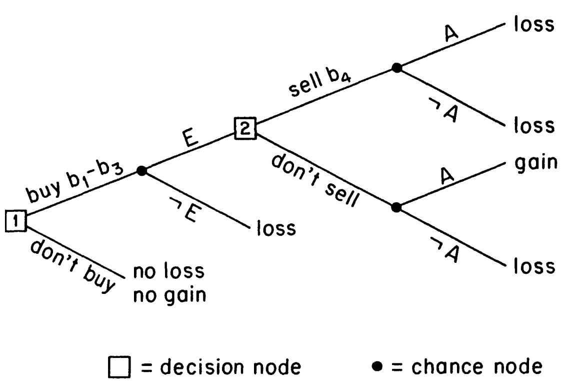

Figure 2.1
Diachronic Dutch book on a decision tree

the decision context this choice leads to a sure loss. She sees also that she gets to node 2 if at node 1 she chooses to buy b_1 to b_3 and E obtains, and further that if she chooses to buy b_1 to b_3 and E fails, she incurs a sure net loss. Thus, all things considered, she sees that buying b_1 to b_3 is unfavorable. It is on just these grounds that Maher (1992) maintains that the diachronic-Dutch-book argument is fallacious (see also Levi 1987).

To the extent that these decision-theoretic considerations are effective in undermining the diachronic-Dutch-book justification for conditionalization, they also bring into question the Dutch-book justification for the axioms of probability. In essence, the decision-theoretic message is to look before you leap. Such advice is just as valid in the synchronic setting as in the diachronic or multitemporal setting. And in the former setting, the advice clashes with the package principle needed in the argument for the principle of additivity of the probabilities of exclusive alternatives, which brings us full circle back to Schick's (1986) objection to Dutch-book arguments. The circle leaves me in an unsettled position. I agree, for example, that if I adopted a rule of belief change other than conditionalization and if I were cagey enough to draw up the decision tree for diachronic Dutch book, then I would refuse to accept the initial bets. But since I regard each of these bets as fair, should I not therefore recognize that there is something amiss in my opinion/preference structure? Grounds for a definitive answer do not exist, or if they do, I do not know of them.

A different and more modest justification for conditionalization has been given by Teller (1976), who argues that there are specifiable circumstances under which it can be maintained that if any change in belief is reasonable, then such a change must be via conditionalization. To identify some of these circumstances, Teller proves the following formal result (see also Teller and Fine 1975). Suppose that $\Pr_{\text{old}}(E) > 0$ and that the agent's domain $\mathscr{A}$ of beliefs is *full* in the sense that for any number q and any $A \in \mathscr{A}$ such that $\Pr_{\text{old}}(A) = r$ and $0 \leqslant q \leqslant r$ there is a $B \in \mathscr{A}$ such that $B \models A$ and $\Pr_{\text{old}}(B) = q$. Suppose further that $\Pr_{\text{new}}(\cdot)$ is such that $\Pr_{\text{new}}(E) = 1$ and that for all $A, B \in \mathscr{A}$ such that $A \models E$ and $B \models E$, if $\Pr_{\text{old}}(A) = \Pr_{\text{old}}(B)$, then $\Pr_{\text{new}}(A) = \Pr_{\text{new}}(B)$. Then $\Pr_{\text{new}}(\cdot) = \Pr_{\text{old}}(\cdot/E)$.

As can easily be verified under the assumption that $\Pr_{\text{old}}(E) = 0$, Teller's crucial condition $C(E)$ is equivalent to $C'(E)$:

$C(E)$ For all $A, B \in \mathscr{A}$ such that $A \models E$ and $B \models E$, if $\Pr_{\text{old}}(A) = \Pr_{\text{old}}(B)$, then $\Pr_{\text{new}}(A) = \Pr_{\text{new}}(B)$.

C′(E) For all $A, B \in \mathscr{A}$ (whether or not they entail E), if $\Pr_{\text{old}}(A/E) = \Pr_{\text{old}}(B/E)$, then $\Pr_{\text{new}}(A) = \Pr_{\text{new}}(B)$.

There are clear cases where we want to impose $C(E)$ or $C'(E)$ for at least some A and B. Thus, let A be the proposition that Dancer will win the Derby, B the proposition that Prancer will win the Derby, and E the proposition that one or the other has won. Suppose that an agent is initially equally confident of A and B. She now learns precisely that E—that and no more. It would seem that, in accord with $C(E)$, she would be unreasonable in these circumstances to adjust her degrees of belief so that Dancer is now preferred to Prancer (or vice versa). But to invoke the formal result, we need to extend the argument to all pairs of initially equally probable propositions entailing E. It is hard to see how this can be done for any particular $\mathscr{A}$ that is sufficiently rich without using reasoning that would apply equally to any $\mathscr{A}$ and would thus abandon the modesty of the approach.

The basis for an immodest justification can perhaps be found in van Fraassen's (1989) result that under the assumption of the fullness of $\mathscr{A}$, $C(E)$ is implied by the requirement that the new probability of any proposition $A \in \mathscr{A}$ is a function solely of the evidence E and the old probability of A. It is well to note, however, that van Fraassen himself would not take such a justification to imply that conditionalization is necessary for rationality, since in his view rationality does not require that belief change follows a preset rule (see van Fraassen 1989 and 1990).

A different motivation for Jeffrey conditionalization starts from the idea that one should make as small a change as possible in one's overall system of beliefs compatible with the shift in those beliefs directly affected by the learning experience. Consider a probability function Pr on $\mathscr{A}$, thought of as giving the probabilities prior to making an observation. Let $\{E_i\}$ be a partition, intended as the locus of belief change, and let Pr* be a measure on $\{E_i\}$ such that $\Pr^*(E_i) > 0$ and $\sum_i \Pr^*(E_i) = 1$, intended to give the new probabilities of the E_i after observation. One would like to extend Pr* to a probability measure Pr** on $\mathscr{A}$ in such a way that Pr** makes as minimal a change as possible in Pr. Relative to several natural distance measures, the probability obtained by Jeffrey conditionalization fits the bill, although for some distance measures it may not do so uniquely (see Diaconis and Zabell 1982).

When the effect of observation is not so simple as to be localizable in a single partition, the method for updating probabilities becomes problematic. Suppose that one's experience results in new degrees of belief for each of the partitions $\{E_i\}$ and $\{F_j\}$. It is not guaranteed a priori that these degrees of belief are mutually coherent in the sense that they are extendible to a full probability on $\mathscr{A}$. A necessary and sufficient condition for the existence of such an extension is supplied by Diaconis and Zabell (1982). Assuming coherence, one could proceed to produce a new probability function by successive Jeffrey conditionalizations on the two partitions. But the order of conditioning may matter. If we denote the results of Jeffrey conditionalizing on $\{E_i\}$ (respectively $\{F_j\}$) by $\Pr_E(\cdot)$ ($\Pr_F(\cdot)$), then the order does not matter in that $\Pr_{EF}(\cdot) = \Pr_{FE}(\cdot)$ just in case $\Pr_F(E_i) = \Pr(E_i)$ and $\Pr_E(F_j) = \Pr(F_j)$ for all i and j.[12] The interested reader is referred to Diaconis and Zabell 1982 and van Fraassen 1989 for more discussion of these and related matters.

While the cumulative weight of the various justifications for conditionalization seems impressive, it should be noted that the starting assumptions of strict and Jeffrey conditionalization are left untouched. The former assumes that learning experiences have a precise propositional content in the sense that there is a proposition E that captures everything learned in the experience, while the latter assumes that if there is no precise propositional content, still the resulting belief changes can be localized to a partition. One or the other of these assumptions is surely correct for an interesting range of cases, but it is doubtful that they apply across the board. And where the doubt is realized, the present form of Bayesianism is silent.

In the remainder of this book I will concentrate on cases where strict conditionalization applies.

Chapter 2

1. Levi (1980) draws a subtle distinction between such temporal or dynamic conditionalization and what he calls confirmational conditionalization. The latter is atemporal in that it is a constraint on the agent's confirmational commitments at time *t*. It requires that the agent relate via conditionalization his current belief states to the hypothetical belief states that can arise by accepting new evidence. Levi himself accepts confirmational conditionalization but rejects temporal conditionalization, whereas Kyburg (1974) rejects both. The views of these authors are recommended to the reader for consideration, but they will not be discussed here.

2. This example is taken from Richard Jeffrey and Brian Skyrms.

3. For other approaches to conditionalization, see Field (1978) and Garber (1980).

4. Thus, if one wishes to be pedantic, a Bayesian probability space is a triple $(\mathscr{W}, \mathscr{A}, \Pr)$, where $\mathscr{W}$ is a set of possible worlds, $\mathscr{A}$ is a set of sentences or propositions, and Pr is a map from $\mathscr{A}$ to $\mathbb{R}$ satisfying the probability axioms.

5. In particular, if A is a tautology, then $\models A$.

6. See, however, Seidenfeld and Schervish 1983 for the problems this causes for Savage and de Finetti.

7. If the maximum odds an agent is willing to take *on* a proposition are less than the minimum odds she is willing to take *against* the same proposition, then Dutch book cannot be made against her. The resulting calculus of belief involves either subadditivity or interval valued degrees of belief. See Smith 1961 and Williams 1976.

8. Howson (1990a) attempts to overcome some of the above mentioned difficulties by focusing on some idealized contexts where there is a natural connection between subjective probabilities and propensities to bet and where the Dutch-book construction is sound. Howson seems to feel that although the conditions needed to run the Dutch-book construction fail outside of these idealized contexts, still "the constraints imposed as a consequence of Dutch Book considerations generalize quite naturally out of these simple contexts, for no other reason than because probability is a general guide, invoked impartially in *all* contexts" (p. 8). I leave it to the reader to evaluate the plausibility of this claim. Tim Maudlin (private communication) has noted that a thoroughgoing Bayesian ought to take into account the probability that the bet will be paid off. But when this is done, the connection between degree of belief and maximum betting odds may be severed. For example, I may assign a low

probability to the proposition that the world will end tomorrow, but since if it does end, I won't have to pay the bet, I am willing to bet against the proposition at any odds.

9. Suppose that an agent's utility U is a function from wealth to $\mathbb{R}$ and that U is twice differentiable. Economists typically assume that rational economic agents display both *non-satiation* and *risk aversion* for all levels of wealth (see Arrow 1971). In terms of U, these assumptions amount respectively to $U'(w) > 0$ and $U''(w) < 0$ for all w. To understand the economic implications of risk aversion, let X be a random variable that represents a risky asset in the minimal sense that X has a nondegenerate probability distribution, and let $E(X) = r^*$. From $U'' < 0$ and Jensen's inequality it follows that $E(U(X)) < U(E(X)) = U(r^*)$. Furthermore, if we define the cash equivalent of the risky asset X to be the amount of cold cash $\$r^{**}$ such that $U(r^{**}) = E(U(X))$, then $r^{**} < r^*$. For again by Jensen's inequality, $U(r^{**}) < U(r^*)$, and the result follows from nonsatiation.

10. Here 'estimate' must be taken in some primitive sense rather than in the probabilistic sense of expected value (see Shimony 1988).

The problem of the appropriate choice of reference class has bedeviled the frequency interpretation from the beginning. In this instance, however, the problem can be finessed (see van Fraassen 1983a).

11. Change of probability via conditionalization $\mathrm{Pr}_{\mathrm{new}}(\cdot) = \mathrm{Pr}_{\mathrm{old}}(\cdot/E)$ is not reasonable, and the Dutch-book arguments for the change reduce to nonsense unless E is the strongest proposition learned (see Mellor 1971).

12. The reader should be aware that Diaconis and Zabell (1982) assume, in effect, that the sensory stimulation relevant to the partition in question determines the probabilities over the elements of the partition independently of all prior experiences. Formally, this means that for any $\{F_i\}$, $\mathrm{Pr}_{FE}(E_i) = \mathrm{Pr}_E(E_i)$. Field (1978) wants to allow that the probabilities attached to the elements of a partition depend not only on sensory stimulation but also on prior probabilities.

13. Burks (1977) tries to seize the first horn.

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