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A PRIMER OF PROBABILITY LOGIC

ERNEST W. ADAMS

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Appendix 1

Coherent Degrees of Confidence: Axiomatics and Rational Betting Odds

1.1 Introduction

The view on the nature of probability that was sketched in section 9.9 was *objectivist* in the sense that what makes statements about probabilities right is that they correspond to a special kind of fact, and it was *pragmatic* in the sense that it is to a DM's practical advantage to have degrees of confidence that correspond to them. Very often, however, it is simply held that there are fundamentally different kinds of probability, among which 'objective' (or possibly 'physical') are thought to be one kind, and 'subjective', or 'epistemic' are thought to be another.[1] The latter are allied to the degrees of confidence that chapter 9 was concerned with, but the theory of this kind of probability is usually developed intrinsically, without reference to the 'external facts' that make them 'right' or 'wrong' (or, perhaps 'better' or 'worse'). This appendix will sketch the basic ideas of two such theories, both of which are widely held, and both of which have the following objectives: (1) To show how it is possible to measure, or attach numerical values to degrees of confidence that are 'in the mind', and (2) To show that these numerical values can consistently be assumed to satisfy the basic laws of probability—the Kolmogorov axioms. Except for brief remarks in section 9.7, these things were taken for granted in chapter 9, but theories that underlie them have been very influential. The first theory is axiomatic, starting with 'self-evident principles' that coherent or 'consistent' qualitative degrees of confidence should satisfy, and then proving that if they conform to these principles they must be numerically measurable in a way that conforms to the Kolmogorov axioms. The other approach considers the way that degrees of confidence influence the odds

[1] Cf. Carnap (1945).

on bets that people will accept, as in section 9.9, but it only argues that it is to bettors' practical advantage to accept bets at odds based on degrees of confidence that are measurable by probabilities that satisfy the axioms, and it does not try to prove that any one set of probabilities that satisfy the axioms is any more 'rational' than another. The axiomatic approach will be outlined in section 2 of what follows, and the betting odds approach will be outlined in section 3. As it happens, both approaches are quite complicated, and only their basic concepts and central results can be sketched here. This appendix will conclude with a brief critique of the two approaches.

1.2 The Axiomatic Approach

Section 9.7 outlined aspects of a theory of the measurement of degrees of confidence and subjective value jointly, but the theories to be commented on here just focus on the former—the measurement of degrees of confidence independent of subjective value. Their basic assumption, usually, is that while a DM may not be directly aware of the numerical value of his degree of confidence in an 'event', say L, that Jane will take logic, he can at least compare this with his confidence in another event, say E, that Jane will take ethics. For purposes of conciseness, if the DM is more confident that Jane will take logic than he is that she will take ethics, we will write L$\gg$E. The axioms of axiomatic approaches state laws that a rational DM's 'confidence ordering relation', $\gg$, should satisfy. Three that are widely regarded as 'self-evident' are as follows:[2]

A1. $\gg$ *orders* the field of events; i.e., for any events ϕ and ψ, not both $\phi\gg\psi$ and $\psi\gg\phi$, and for any events ϕ, ψ, and η, if $\phi\gg\psi$ then either $\phi\gg\eta$ or $\eta\gg\psi$.[3]

A2. If T is a logical truth and F is a logical falsehood, then T$\gg$F, and for any event ϕ, it is not the case that F$\gg\phi$.[4]

A3. If η is logically inconsistent with ϕ and with ψ then $(\phi\vee\eta)\gg(\psi\vee\eta)$ if and only if $\phi\gg\psi$.

[2]There is a long history of axiomatizations of this kind, and axioms A1–A3 below are essentially due to de Finetti (1937). Chapter 5 of Krantz et. al. (1971) gives a succinct summary of this history, as well as a precise and sophisticated exposition of the theory of qualitative probability representations. The full set of axioms to follow are very close to ones stated in section 5.2 of Krantz et. al., op. cit.

[3]This ordering axiom is less familiar than more commonly stated ordering conditions that include transitivity, i.e., that if $\phi\gg\psi$ and $\psi\gg\eta$ then $\phi\gg\eta$. However, the latter is easily seen to follow from axiom A1. A field of 'events' may be considered to be a class of propositions, ϕ, ψ, etc., where if ϕ and ψ are in the class then so are $\sim\phi$, $\phi\&\psi$, and $\phi\vee\psi$, and these are assumed to satisfy the usual logical laws.

[4]Note the dependence of the qualitative degree of confidence axioms on underlying deductive concepts. That the DM is assumed to be able to recognize logical truths and falsehoods is one of the idealizations involved in the theories that we are concerned with.

These axioms are necessary for the existence of a *probability representation*, i.e., they are necessary to guarantee the existence of a probability function, p( ), that satisfies the condition that $p(\phi)>p(\psi)$ should hold if and only if $\phi\gg\psi$. They are commonly postulated without argument, since it is assumed to be self-evident that any rational DM's degrees of confidence should conform to them.[5] However Kraft, Pratt, and Seidenberg (1959) showed that the three axioms aren't sufficient by themselves to guarantee the existence of a probability representation,[6] and subsequent theoretical work has tended to supplement A1–A3 with other conditions, which are not strictly necessary for probabilistic representability, but which together with A1–A3 are sufficient for it.[7] Two axioms of that kind are given in section 5.2.3 of Krantz et. al. (1971), rough and simplified versions of which are as follows:

A4. The field of events is *Archimedean*, i.e., if $\phi\gg F$ then there cannot be an infinite number of disjoint 'replicas' of ϕ, say $\phi_1,\phi_2,\ldots$, each of which is equivalent to ϕ in the sense that for any i=1,2,... it is not the case that $\phi\gg\phi_i$ or $\phi_i\gg\phi$.

A5. For any $\phi\gg\psi$, there exists some ψ' that logically entails ϕ, which is equivalent to ψ in the above sense.

Given A1–A5, we can state:

Theorem 1. If a field of events satisfies axioms A1–A5 then there exists a probability function $p(\phi)$ defined for all ϕ in the field such that $\phi\gg\psi$ holds if and only if $p(\phi)>p(\psi)$.[8]

It remains to comment briefly on the status of axioms A4 and A5, which are much less self-evident than A1–A3. Adams (1992) has discussed this question in detail, especially as concerns the question of whether axioms A4 and A5 have testable 'empirical content' in the sense that there might be 'data' the form $\phi\gg\psi$ that are consistent with the existence of a probability representation, but which conflict with these axioms. Trivially, the

[5]But some writers have tried to justify certain of them. E.g., Davidson and Suppes (1956) have given an interesting argument for the transitivity of subjective orderings. Other authors, e.g., Chihara (1987), have worried about the idealization involved in assuming that DMs can recognize deductive properties and relations.

[6]See Krantz et. al., op. cit., section 5.2.2. General results of Scott and Suppes (1958) are easily adapted to show that no purely universal axioms (roughly, axioms that don't postulate the *existence* of events of special kinds) can be necessary and sufficient for 'probabilistic representability'.

[7]Scott (1964) is an exception, since it presents an infinite set of purely universal axioms that are jointly sufficient to guarantee the existence of a probability representation for any finite field. These axioms are essentially the same as ones worked out and discussed independently in Adams (1965).

[8]Cf. Theorem 2 on p. 208 of Krantz et. al. (1971). Section 5.6 of that work sets forth the results of R.D. Luce on the representation of qualitative conditional probability orderings.

combined axioms A4 and A5 have empirical content, but it is shown that an Archimedean condition akin to A4 does not. In a sense it is 'purely technical': it can be assumed 'for technical reasons' in the proof of a 'probability representation theorem', and we are assured *a priori* that no data that are consistent with the existence of such a representation will be inconsistent with this axiom.[9] But the justification of axioms like A5 raises deep questions of scientific method that cannot be pursued here.

Now we turn to the betting odds approach to the 'logical laws' of degrees of confidence.

1.3 Rational Betting Odds and Dutch Books

The discussion to follow will concentrate mainly on concrete examples of the behavior that theories of rational betting apply to, and it will enter into very few mathematical details. For an excellent elementary exposition of the mathematical theory the reader is referred to sections VI.4–VI.7 of Skyrms (1986).

Consider the following situation. A *bookie*, i.e., a person who takes bets at stated odds on the outcomes of horse races, posts the following odds in bets of different amounts on horses X, Y, and Z *placing*, i.e., he announces that he will accept bets at those amounts on their coming in first or second, in a three-horse race between them. Specifically, he will accept \$8 bets that pay \$10 if horse X places in the race, \$6 bets that pay \$10 if Y places, and \$5 bets that pay \$10 if Z places.[10]

It would not be unreasonable for a bookie to post odds like the ones stated above on any one of the three 'events' of horses X, Y, and Z placing in the race. For instance, if he knew that horse Z was a very poor runner in comparison to X and Y, it would be quite reasonable for him to post odds of less than 50% that Z would come in first or second in the race. But now we come to a crucial question: admitting that situations can be imagined in which it would be reasonable for the bookie to post the odds described above for bets on any one of the three events, does it follow that there are situations in which it would be reasonable to post these odds on

[9] However, Manders (1979) examines this and points out the possibility of inductively confirming the existence of an infinite set of data that does this.

[10] Odds are more commonly given, e.g., as "odds of 4:1" that a given horse will win a race, which would imply that a bettor who placed a \$1 bet that X would win, which pays off \$5 if X does win, would come out a net \$4 richer in that case. 4:1 is the ratio between the bettor's net gain of \$4 if he wins on the bet and his net loss of \$1 if he loses. Characterizing odds as is done here, in terms of \$2 that a bettor must place on a bet that wins \$10, is done for two reasons. (1) It makes explicit the amounts of money involved in the transactions both of the bettor paying the \$2 to place the bet, and of his being paid \$10 if he wins. (2) The ratio of these two amounts, namely \$2/\$10 = .2, is more directly related to probabilities than the four-to-one ratio involved in the more common way of stating odds.

all three of them? If he did then a bettor could place bets on the three events simultaneously, or, as it is sometimes said, she could *make a book*, and what we are asking is whether it would be rational for the bookie to allow this. In fact, there is a simple argument that this would be highly irrational.

If the bettor made three bets simultaneously, that horse X would place, that horse Y would place, and that horse Z would place, he would pay a total of \$8+\$6+\$5 = \$19 making them. But clearly she would have to win two of the three bets, since in a race just between three horses only the horse finishing last wouldn't place. Therefore no matter which horses placed, the bettor would win a total of 2×\$10 = \$20. Hence she would end up winning \$20−\$19 = \$1 more than she paid out, and the bookie would end up \$1 worse off than he would have been if he had never accepted the bets.

A combination of bets in which a bettor necessarily ends up with an overall net gain and a bookie ends with a overall net loss, no matter what the results of the individual bets are, is called a *Dutch Book*,[11] and it seems irrational for bookies to post odds that would allow bettors to make such books. This leads us to inquire into the laws of 'coherent' betting odds, i.e., ones that would not allow bettors to make Dutch Books. A rough and ready argument to be given below relates these laws to laws of rational degrees of confidence, but first we will see that the laws of rational betting odds are themselves related to the laws of probability in a very simple way.

The Dutch Book described above is made possible by the following curious fact: The sum of the odds of 8 in 10, 6 in 10, and 5 in 10, namely .8+.6+.5, equals 1.9, and this is less than the sum of any possible probabilities of X placing, of Y placing, and of Z placing. This is made clear in the following table, which lists the bets on X, Y, and Z placing (as well as three other bets which will be returned to later), and which enumerates all of the possible orders in which X, Y, and Z might finish the race, with their probabilities. Thus, the probability of the horses finishing with X first, Y second, and Z last, abbreviated as X>Y>Z, is given as 'a', the probability of their finishing in the order X first, Z second, and Y last, abbreviated as X>Z>Y, is given as 'b', and so on. Then the probability of X placing is the sum of the probabilities of the horses finishing in any of the orders

[11]A variant, called semi-Dutch Book in Shimony (1970) is a system of bets on which a bettor cannot suffer a net loss, no matter what the outcomes of the events bet on are, but in which some possible outcomes yield a net gain. For example, bets in which the bettor pays \$0 and wins \$10 if horse X places cannot lose, but it can win \$10 if X actually does place. This appendix focuses on 'strong Dutch Books', but Adams (1962) also presents necessary and sufficient conditions for systems of betting odds not to allow semi-Dutch Books, i.e., which exclude systems that are called weakly rational in the paper.

X>Y>Z, X>Z>Y, Y>X>Z, and Z>X>Y, which is a+b+c+e. Similarly, you can see from the table that the probability of Y placing is a+c+d+f and the probability of Z placing is b+d+e+f:

		events					
possible finishes	distri-bution	X places	Y places	Z places	X wins	X last	if X doesn't win he places
X>Y>Z	a	yes	yes	no	yes	no	no bet
X>Z>Y	b	yes	no	yes	yes	no	no bet
Y>X>Z	c	yes	yes	no	no	no	bet wins
Y>Z>X	d	no	yes	yes	no	yes	bet wins
Z>X>Y	e	yes	no	yes	no	no	bet loses
Z>Y>X	f	no	yes	yes	no	yes	bet loses
probabilities→		a+b+c+e	a+c+d+f	b+d+e+f	a+b	d+f	(c+d)/(c+d+e+f)

Given the above, it follows that

$$\begin{aligned} &p(\text{X places})+p(\text{Y places})+p(\text{Z places}) \\ &= (a+b+c+e)+(a+c+e+f)+(b+d+e+f) \\ &= 2(a+b+c+d+e+f) = 2, \end{aligned}$$

and, as stated previously, that is greater than 1.9, which is the sum of the odds. This implies in turn that at least one of the probabilities of X placing, Y placing, and Z placing must be greater than the corresponding odds. The following theorem shows the importance of this:

Theorem 2. If in any possible system of probabilities that a set of 'events' might have, there is at least one event whose probability is greater than the odds posted on a bet on it, then it is possible to make a Dutch Book betting at these odds.[12]

The converse also holds:

Theorem 3. If the odds posted for bets on events in some system of events are no greater than some probabilities that these events might have, then it is not possible to make a Dutch Book betting at these odds.

For example, suppose that instead of posting odds of \$8, \$6, and \$5, respectively, to win \$10 on horses X, Y, and Z placing, the bookie had

[12] 'Classical' Dutch Book theorems are due to de Finneti (1937), Kemeny (1955), Lehman (1955), Shimony (1955) and (1970), and others. Chapter IV of Vickers (1976) gives very elegant proofs of the central classical results. More exact versions of the theorems stated here are given in Adams (1962) and they generalize the classical theorems in two important ways: (1) They allow the bookie to restrict the bets she will accept, e.g., she isn't necessarily 'fair', and (2) They consider the possibility of justifying the countable additivity axiom by reference to infinite sets of bets. Recent unpublished papers of Stinchcombe and of Spielman examine this axiom from the same point of view. It should be noted that the present theorems hold only provided that the bettor is allowed to place any number of bets on any of the events at the posted odds. An example that follows shows that it is sometimes necessary to place more than one bet on a single event in order to make a Dutch Book.

posted ones of $8, $6, and $6 or .8, .6, and .6, respectively, to win $10 on these events. These odds actually equal the probabilities that the events in question would have if they were generated by a probability distribution in which a = b = c = e = .2 and d = f = .1, hence according to theorem 2 the bettor couldn't make a Dutch Book betting at these odds. Similarly, if the odds were $9, $7, and $7 to win $10, or .9, .7, and .7, on X, Y, and Z placing, they would actually be greater than the probabilities of .8, .6, and .6 generated by the above distribution, and therefore the bettor couldn't make at Dutch Book betting at these odds either.

It follows from theorems 1 and 2 that if a bookie is 'rational' then the betting odds that he posts on events like the results of a horse race should be at least as great as the probabilities that these events could have, assuming that these probabilities satisfy the axioms of probability. Of course, this doesn't prove that the odds should *equal* probabilities that the events might have, and clearly odds on X's, Y's, and Z's placing of .9. 7, and .6, as above, *couldn't* equal any possible probabilities. In fact a bookie who depended on his winnings to make a livelihood would be foolish to post odds that did equal probabilities, because if he did, his income in the long run couldn't be expected to exceed the amounts that he would have to pay out to persons who won their bets.[13]

One way of forcing the bookie to post odds that equal probabilities that has often been suggested is to require the odds to be 'fair', in the sense that he should be willing to bet at either 'end' of the odds.[14] For instance, if he posts odds that horse X will place that pay $10 on an $8 bet, he should also be willing to make a bet of $8 to win $10 if X places. If fairness is required it is easy to see that the only way the bookie can avoid a Dutch Book is to post odds that actually equal some possible probabilities that the events in question might have. These odds can be related in turn to degrees of confidence.

Sometimes it is assumed that fair betting odds *are* rational degrees of confidence, and assuming this the Dutch Book theorems would guarantee that rational degrees of confidence should satisfy the laws of probability. But this is too simple. At best we can argue that a bookie's degrees of confidence should *influence* the odds that he posts, not that they should determine them. Clearly there are other influences as well, such as his need

[13]This related to the fact that in parimutuel race track betting the state skims a percentage 'off the top', so that the 'bookie' always pays out a little less than he would if he posted 'fair' odds.

[14]Skyrms (1986: 186–9) defines fair bets differently. Adams (1962: 28) presents an argument that odds should be close to 'fair probabilities', as those are defined here. If there are several bookies and one of them posts odds that are significantly higher than such probabilities, then rivals can attract the first bookie's 'customers' away, by posting odds that are closer to fair probabilities, while still avoiding the possibility of a Dutch Book.

to gain a livelihood. In fact, as will be commented on in more detail in the concluding section, deciding to post betting odds and choosing which odds to post can be viewed as decision problems of the kind discussed in chapter 9, and when viewed this way, many complications emerge which can 'distort' the relation between the odds and degrees of confidence. But there is a plausible argument that the distortion can be eliminated by insisting on fairness. The argument goes as follows.

Suppose we assume that a bookie's betting odds are determined jointly by his degrees of confidence and his 'mercenary' need to gain a livelihood. The latter requires him to take in more money 'on the average' on bets that he accepts than he pays out on bets that win, and that requires him to post unfair odds. Conversely, it is suggested that the odds he would post if he were required to be fair would be independent of this kind of mercenary motive, and they should only depend on his degrees of confidence. If so, they should *measure* his degrees of confidence, i.e., the greater his confidence in the occurrence of an event is, the higher the odds he posts on it should be. And this is just what we wanted to show, because we have argued that fair odds must satisfy the laws of probability. A generalization of this has important implications.

Suppose that, rather than posting odds on X, Y, and Z's placing, the bookie posted odds as follows: (1) \$5 to win \$10 on X actually winning the race, (2) \$2 to win \$10 on X finishing last, and (3), \$5 to win \$10 on the contingent possibility of X at least placing, even if he doesn't win. In bet (3) it is assumed that if X does win the bettor's money is refunded, so that, in effect, bet (3) is a *conditional bet.*[15] Would these odds be rational? In fact, no, because the bettor could make a Dutch Book by simultaneously placing one bet of \$5 that X would win, two bets of \$2 that X would finish last, and two bets of \$5 that if X didn't win he would place. Then the total amount that the bettor would bet would be $1\times\$5+2\times\$2+2\times\$5 = \$19$, but her 'net bet' would only be \$9 if X won the race, because in that case she would get back the amount she put down on the two bets of \$5 that if X *didn't* win he would place.

Now consider what the result would be in the three cases of X finishing first, finishing second, and finishing last. In the first case, in which the bettor's net bet would be \$9, she would win \$10 on the bet that X *would* win the race, so she would end up \$1 ahead. In the second case her net bet would be \$19, but now she would win \$10 on each of the two bets that she made that if X didn't win he would at least place, so she would end up with a \$1 net win. In the last case her net bet would also be \$19, but in this case she would win \$10 on each of her bets that X would finish

[15]These are similar to the bets that if coins are tossed they will fall heads, that were commented on in the final part of section 9.9⋆.

last, and again she would end up with a \$1 net gain. In other words the bettor would make a Dutch Book, because she would come out \$1 ahead no matter what the outcome of the race was. That this is possible follows from the next theorem, which combines and generalizes theorems 1 and 2:

Theorem 4. Suppose that odds of o(e) are posted on bets on events e and/or odds o(e|c) are posted on e, contingent on c being the case. Then it is possible to make a Dutch Book betting at these odds if and only if for every system of probabilities that these events might have there is at least one event e such that p(e) > o(e), or one event e contingent on c being the case such that p(e|c) > o(e|c).[16]

Thus, rational odds, o(e|c), on 'conditional events' are related to conditional probabilities, p(e|c), in the same way that odds on unconditional events, o(e), are related to the unconditional probabilities, p(e). And, assuming fairness, we can argue that these odds should measure the degrees of confidence that a bookie has in propositions that describe conditional events, such as the ones discussed in section 9.8, that are involved in decision making in nonindependence cases.

Similar considerations have been advanced to justify the assumption that degrees of confidence should change over time in accordance with Bayes' principle.

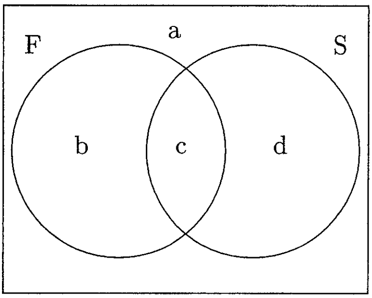
1.4 Dynamical Dutch Books

Consider the following variant of the conditional betting example. A coin will be tossed twice, and a bookie takes the following three actions: (1) He posts odds of \$5 to pay \$10 if the coin falls heads on the first toss, (2) He posts odds of \$2 to pay \$10 if the coin doesn't fall heads on either toss, and (3) He promises that if the coin doesn't fall heads on the first toss, he will post odds of \$5 to win \$10 if the coin falls heads on the second toss. Now, a variant of the bettor's earlier strategy allows her to make a 'dynamic Dutch Book': (1a) Before the coin is tossed the first time, she places one bet of \$5 at the bookie's odds (1), that the coin will fall heads on the first toss, (2a) Also before the coin is tossed the first time, she places two bets of \$2 at the bookie's odds (2), that the coin won't fall heads on either toss, and (3a) She decides that if the coin doesn't fall heads on the first toss, she will place two bets that it will fall heads on the second toss, at the odds (3), that the bookie has promised he would post if the coin doesn't fall heads on the first toss. Readers can work it out for themselves that if the bettor adheres to this strategy and the bookie abides by his promise that if

[16]This is also proved in Adams (1962), subject to the proviso that the bettor can place any number of bets at the given odds, but with the condition that p(e|h) > o(e|h) replaced by p(e&c) > o(e|c)×p(c). This sidesteps the problem of defining p(e|h) when p(h) = 0.

the coin doesn't fall heads on the first toss, he will post odds (3), then the bettor must end with a net gain of \$1, no matter what the results of the individual tosses are. Thus, the bettor has made a Dutch Book, which is 'dynamical' because it involves actions taken at two different times: odds posted and bets made before the coin is tossed the first time, and, if it doesn't fall heads the first time, odds offered and bets placed before it is tossed the second time.

The following diagram will help to 'diagnose' the problem above:



F = the coin falls heads on the 1st toss
S = the coin falls heads on the 2nd toss
$o(F) = .5 \geq p_0(F) = b+c$
$o(\sim(F\vee S)) = 2 \geq p_0(\sim(F\vee S) = a$
$a/d \leq .2/.3$
$p_0(S|\sim F) = \frac{1}{1+(a/d)} \geq \frac{1}{1+(.2/.3)} = .6$

The probabilities given above, $p_0(F)$, $p_0(\sim(F\vee S))$, and the conditional probability $p_0(S|\sim F)$, are ones that obtain prior to the first coin-toss, and the first two are assumed to be 'rational' in the sense that they are not greater than the corresponding odds, $o(F) = .5$ and $o(\sim(F\vee S)) = .2$. These assumptions entail that $p_0(S|\sim F) \geq .6$, as shown. Now, if Bayes' principle is assumed then $p_0(S|\sim F)$ should equal $p_1(S)$, the unconditional probability of S *posterior* to learning that the coin didn't fall heads on the first toss. Thus, $p_1(S)$ should be at least .6. But that *shouldn't* be greater than the odds of .5 that the bookie promised to post on the coin's falling heads on the second toss, after it is learned that it didn't fall heads on the first toss. Generalizing, it seems that to avoid the possibility of a dynamic Dutch Book, a bookie should post odds that are at least as high as probabilities that conform to Bayes' principle, and to be fair he should post ones that equal probabilities conforming to the principle.

Thus Dutch Book considerations not only seem to justify the conclusion that degrees of confidence should satisfy the 'static' laws of probability, but they should also evolve in accord with Bayes' principle. However, this is not quite decisive.

1.5 Critique

One objection to the Dutch Book argument for Bayes' principle is that it assumes that the bookie promises in advance that if certain things happen he will then accept bets at predetermined odds, e.g., if the coin doesn't fall heads on the first toss then he will accept bets of \$5 that pay \$10 if it falls heads on the second one. But for the bookie to bind himself *a priori* in this way is in effect to post odds *a priori*, on the conditional bet that if the coin doesn't fall heads on the first toss it will fall heads on the second. All the argument shows, therefore, is that it would be incoherent for the bookie to post odds on three bets, including a conditional one, *a priori*. But theorem 3 already told us this, and there is no necessary connection between that and changes in betting odds. What we must ask is whether the bookie should be bound to accept bets placed *a posteriori*, at the odds that he posts on conditional bets placed *a priori*. That seems doubtful.

It is obvious that whatever odds a bookie posts, he only commits herself to accept bets at those odds during a limited period of time. For instance, it would be irrational for him to accept bets on a race that are placed after the race is over—even more so than to post odds that permitted a Dutch Book. In fact, this is illustrates a point made earlier, that the odds that a bookie posts and the bets that he accepts are generally influenced by other things besides his degrees of confidence. That he wouldn't accept bets placed 'after the fact' reflects the fact that he is influenced by what he thinks bettors know. In particular, he knows that he ought to know at least as much about the events he posts odds on as bettors do when they place bets at those odds, and the reason he doesn't accept bets placed after the fact is because he knows that the persons who placed them then would know more than he did when he posted the odds.

The same considerations even call into question the thesis that 'static' betting odds ought to conform to the probability axioms. If no restrictions were placed on the times at which a bettor could make bets at a bookie's odds, say on the outcome of a horse race, she could always make sure of winning by waiting until the race was over before placing her bets. If the bookie had to accept such bets, the only way that he could avoid a sure loss would be to post odds of $10 to win $10 on all events, which would be inconsistent with the laws of probability, and would be in effect to accept no bets at all! Moreover, if he were compelled to be 'fair', and post odds that satisfied the laws of probability, he would have to offer ones that differed markedly from his degrees of confidence in order to minimize his losses. For instance, if he were 10% confident that horse X would win the race and 90% confident that he wouldn't, he should post odds of $2.50 to win $10 that X would win the race, and $7.50 to win $10 that he wouldn't win. If he did this and allowed bettors to wait until the race was over before placing their bets, his expected loss would be $6 per bet. While, if he posted odds equalling his degrees of confidence, his expected loss would be $10 per bet.[17]

[17]This and other 'oddities' of the relation between degrees of confidence and betting odds that come out when then choice of odds is viewed as a decision problem in the manner

But, to conclude, approaching the subject of consistency or coherence in degrees of confidence either axiomatically or via rational betting odds seems very dubious to the writer. What DMs and bookies *want* are results that are beneficial to themselves, and consistency should be valued primarily to the extent that it tends to such results. As F.P. Ramsey wrote "It is better to be inconsistent and right some of the time than to be consistent and wrong all of the time" ("Truth and Probability," p. 191).[18] Assuming this, if a DM's degrees of confidence conform to the laws of probability, this is most likely to be because he attempts to bring them into correspondence with objective probabilities such as those discussed in section 9.9, and these are what necessarily satisfy the axioms or probability.

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of chapter 9 are discussed section 2 of Adams and Rosenkrantz (1980). The special precautions that must be taken to assure that bettors are no more knowledgeable than bookies are discussed in Hacking (1967).

[18]Adams (1988a) points out a secondary virtue of consistency: namely to facilitate communication. Blatant contradiction generally produces confusion in a hearer, thus defeating speakers' purposes.

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